Chapter 3: Read Theorem 3.4. Do 9, 10, 11, 12.

Additional homework (suggestion: do these before the book homework):

1. Let $U, V, W$ be vector spaces over $\mathbb{F}$ and let $A \in \mathcal{L}(V, W)$. Determine whether the function $T : \mathcal{L}(U, V) \to \mathcal{L}(U, W)$ defined by $T(X) = AX$ is a linear map.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map defined by
   \[ T(x, y) = (2x - y, -8x + 4y). \]
   (a) Find the null space of $T$.
   (b) Find the range of $T$.
   (c) Find a basis for the null space of $T$.
   (d) Find a basis for the range of $T$.

3. Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be the linear map defined by
   \[ T(x_1, x_2, x_3, x_4, x_5) = (x_1 + 3x_2 - 2x_3 - 3x_4, x_3 + 2x_4 + 3x_5, x_5, 2x_5). \]
   (a) Find the null space of $T$.
   (b) Find the range of $T$.
   (c) Find a basis for the null space of $T$.
   (d) Find a basis for the range of $T$.

4. Let $A$ and $B$ be nonempty sets and let $f : A \to B$ be a function. Prove
   (a) $f$ is injective if and only if there exists a function $g : B \to A$ such that $g \circ f = \text{id}_A$.
   (b) $f$ is surjective if and only if there exists a function $g : B \to A$ such that $f \circ g = \text{id}_B$. 
