MTH 309

Additional Problems for Sec 2.3

1. For each of the following functions, determine whether the function is one to one.

   (a) $f : \{1, 2, 3\} \to \{2, 3, 4\}$
   $f(1) = 3, \quad f(2) = 4, \quad f(3) = 4$

   (b) $f : \mathbb{N} \to \mathbb{N}$
   $f(n) = n + 3$

   (c) $f : \mathbb{Z} \to \mathbb{Z}$
   $f(n) = n + 3$

   (d) Let $\mathcal{A} = \{0, 1\}$ and let $\mathcal{A}^*$ be the set of bit strings.
   $f : \mathcal{A}^* \to \mathbb{N}$
   $f(w) =$ the number of 1's in w

   (e) $f : \mathcal{P}(\{0, 1, \ldots, n\}) \to \mathcal{P}(\{1, 2, \ldots, n + 1\})$
   $f(S) = \{x \in \{1, 2, \ldots, n + 1\} \mid x - 1 \in S\}$

   (f) $f : \mathcal{P}(\{1, 2 \ldots, n\}) \to \mathcal{P}(\{1, 2, \ldots, n + 1\})$
   $f(S) = S \cup \{n + 1\}$

2. For each of the following bijections, find its inverse. (Be sure to include domain, codomain and rule.)

   (a) $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = 7x - 4$.

   (b) Let $\mathcal{A} = \{0, 1\}$. $f : \mathcal{A}_n \to \mathcal{A}_n$, where $f(w_1 w_2 \ldots w_n) = w_n w_1 w_2 \ldots w_{n-1}$.

   (c) $f : \{T \in \mathcal{P}(\{1, 2, \ldots, n\}) \mid n \in T\} \to \mathcal{P}(\{1, 2, \ldots, n - 1\})$, where $f(T) = T - \{n\}$.

3. Let $0 \leq k \leq n$. Find a bijection from the set of subsets of size k of the set $U = \{1, 2, \ldots, n\}$ to the set of subsets of size $k+1$ of the set $V = \{1, 2, \ldots, n+1\}$ that contain the integer $n + 1$. (Describe the domain and codomain by using set builder notation and express the rule by using unions.)