

MTH 309

Additional Problems for Sec 2.3

1. Let $\mathcal{A} = \{0, 1\}$ and let \mathcal{A}^* be the set of bit strings.

$$f : \mathcal{A}^* \rightarrow \mathbb{N}$$

$$f(w) = \text{the number of 1's in } w$$

Find $f(110100)$.

2. $f : \mathcal{P}(\{0, 1, \dots, n\}) \rightarrow \mathcal{P}(\{1, 2, \dots, n+1\})$

$$f(S) = \{x \in \{1, 2, \dots, n+1\} \mid x-1 \in S\}$$

Find

(a) $f(\{2, 5, 9\})$

(b) $f(\emptyset)$

(c) $f(\{0, 1, \dots, n\})$

3. $f : \mathcal{P}(\{1, 2, \dots, 20\}) \rightarrow \mathcal{P}(\{1, 2, \dots, 21\})$

$$f(S) = S \cup \{21\}.$$

Find

(a) $f(\{2, 5, 9\})$

(b) $f(\emptyset)$

(c) $f(\{1, \dots, 20\})$

4. Let $f : \mathcal{P}(\{1, 2, \dots, n\}) \rightarrow \{0, 1\}_n$

$$f(T) = w_1 w_2 \cdots w_n \text{ where } w_i = \chi(i \in T) \text{ for all } i \in \{1, 2, \dots, n\}.$$

For $n = 7$, find

(a) $f(\{2, 4, 5\})$

(b) $f(\emptyset)$

(c) $f(\{1, 2, 3, 4, 5, 6, 7\})$

5. Decide which of the following are functions.

(a) $f : \{1, 2, 3\} \rightarrow \{2, 3, 4\}$

$$f(1) = 3, \quad f(2) = 4, \quad f(3) = 4$$

- (b) $f : \mathbb{N} \rightarrow \mathbb{N}$
 $f(n) = n + 3$
- (c) $f : \mathbb{Z} \rightarrow \mathbb{Z}$
 $f(n) = n + 3$
- (d) $f : \mathbb{N} \rightarrow \mathbb{N}$
 $f(n) = n - 3$
- (e) Let $\mathcal{A} = \{0, 1\}$ and let \mathcal{A}^* be the set of bit strings.
 $f : \mathcal{A}^* \rightarrow \mathbb{N}$
 $f(w) = \text{the number of 1's in } w$
- (f) $f : \mathbb{Q} \rightarrow \mathbb{Q}$
 $f(p/q) = p$
- (g) $f : \mathcal{P}(\{0, 1, \dots, n\}) \rightarrow \mathcal{P}(\{1, 2, \dots, n + 1\})$
 $f(S) = \{x \in \{1, 2, \dots, n + 1\} \mid x - 1 \in S\}$
- (h) $f : \mathcal{P}(\{1, 2, \dots, n\}) \rightarrow \mathcal{P}(\{1, 2, \dots, n + 1\})$
 $f(S) = S \cup \{n + 1\}$
- (i) $f : \mathbb{Z}^+ \rightarrow \mathbb{Q}$
 $f(n) = 1/n$