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ERRATUM: GLOBAL STABILITY IN CHEMOSTAT-TYPE COMPETITION MODELS WITH NUTRIENT RECYCLING*

SHIGUI RUAN† AND XUE-ZHONG HE‡

Abstract. This note corrects some typos and errors in the paper [S. Ruan and X.-Z. He, *SIAM J. Appl. Math.*, 58 (1998), pp. 170–192].

Key words. competition, global stability, nutrient recycling

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The main error in the paper [1] is that conditions on the matrix B should be replaced by conditions on the matrix AB at four separate places.

1. On page 177, condition (iii) in Theorem 2.8 should read

(iii) *the matrix AB is semipositive definite.*

2. On page 182, there were a few typos in Theorem 3.8 and an error in condition (v). The corrected version of the theorem is as follows.

THEOREM 3.8. *Assume that*

(i) *system* (3.1) *has a positive equilibrium* $E^* = (S^*, N_1^*, N_2^*)$;

(ii) $D + D_i < m_i$, $b_iD_i < \mu_i p(S_i^*)$, $i = 1, 2$;

(iii) $T_f < \infty$, $T_i^* = (1/d_i^*) \int_0^\infty F(s)[e^{d_i^* s} - 1]\, ds < \infty$ *with* $d_i^* := (D + D_i) + \sum_{j=1}^2 \delta_{ij} m_j$, $i = 1, 2$;

(iv) $b_iD_i[(m_i + \sum_{j=1}^2 \delta_{ij} N_j^*)T_i^* + m_i T_f]/2 < \mu_i$, $i = 1, 2$;

(v) *The matrix AB is semipositive definite, where* $A = \mathrm{diag}(\alpha_1, \alpha_2)$ *with* $\alpha_i = [\mu_i p(S_i^*) - b_iD_i]/m_i$ $(i = 1, 2)$ *and* $B = (b_{ij})_{2\times 2}$, $b_{ij} \geq 0$ *defined by*

$$b_{ij} = \begin{cases} \delta_{ii} - \frac{T_f m_i}{2[\mu_i p(S^*) - b_i D_i] N_i^*} \sum_{j=1}^2 b_j D_j \delta_{ji} m_j & \text{if } i = j, \\ \delta_{ij} & \text{if } i \neq j. \end{cases} \tag{3.25}$$

Then E^* is global asymptotically stable.

3. On page 186, condition (iii) in Theorem 4.3 should read

(iii) *the matrix AB is semipositive definite.*

4. On page 188, Theorem 4.6 should read as follows.

THEOREM 4.6. *Assume that*

(i) *system* (4.11) *has a positive equilibrium* $E^* = (S^*, N_1^*, \ldots, N_n^*)$;

(ii) $D + D_i < m_i$, $b_iD_i < \mu_i p(S_i^*)$, $i = 1, 2, \ldots, n$;

(iii) $T_f < \infty$, $T_i^* = (1/d_i^*) \int_0^\infty F(s)[e^{d_i^* s} - 1]\, ds < \infty$, $i = 1, 2, \ldots, n$, *where* $d_i^* = (D + D_i) + \sum_{j=1}^n \delta_{ij} m_j$;

(iv) $b_iD_i[(m_i + \sum_{j=1}^n \delta_{ij} N_j^*)T_i^* + m_i T_f]/2 < \mu_i$, $i = 1, 2, \ldots, n$;

(v) *the matrix AB is semipositive definite, where* $A = \mathrm{diag}(\alpha_i)_{n\times n}$ *with* $\alpha_i =$

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†Department of Mathematics, University of Miami, P.O. Box 249085, Coral Gables, FL 33124-4250 (ruan@math.miami.edu).

‡School of Finance and Economics, University of Technology, P.O. Box 123, Broadway NSW 2007, Sydney, Australia (tony.he1@uts.edu.au).

$[\mu_i p(S_i^*) - b_i D_i]/m_i$ $(i = 1, \ldots, n)$ and $B = (b_{ij})_{n\times n}$ with $b_{ij} \geq 0$ defined as follows:

$$b_{ij} = \begin{cases} \delta_{ii} - \frac{T_f m_i}{2[\mu_i p(S^*) - b_i D_i] N_i^*} \sum_{j=1}^n b_j D_j \delta_{ji} m_j & \text{if} \quad i = j, \\ \delta_{ij} & \text{if} \quad i \neq j. \end{cases}$$

Then E^* is globally asymptotically stable.

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REFERENCE

[1] S. Ruan and X.-Z. He, *Global stability in chemostat-type competition models with nutrient recycling*, SIAM J. Appl. Math., 58 (1998), pp. 170–192.