Isaac Newton

De Methodus Fluxionum et Serierum infinitorum, written 1664–1671. Methods of Series and fluxions, in D. T. Whiteside, *The Mathematical papers of Isaac Newton*, Cambridge: Cambridge University Press, vol. III (1670–1673), 1969, pp. 43–47.

The reduction of affected equations

When, however, affected equations are proposed, the manner in which their roots might be reduced to this sort of series should be more closely explained, the more so since their doctrine, as hitherto expounded by mathematicians in numerical cases, is delivered in a roundabout way (and indeed with the introduction of superfluous operations) and in consequence ought not to be brought in to illustrate the procedure in species. In the first place, then, I will discuss the numerical resolution of affected equations briefly but comprehensively, and subsequently explain the algebraical equivalent in similar fashion.

Let the equation $y^3 - 2y - 5 = 0$ be proposed for solution and let the number 2 be found, one way or another, which differs from the required root by less than its tenth part. I then set $2 + p = y$, and in place of y in the equation I substitute $2 + p$. From this there arises the new equation $p^3 + 6p^2 + 10p - 1 = 0$, whose root p is to be sought for addition to the quotient. Specifically, (when $p^3 + 6p^2$ is neglected because of its smallness) we have $10p - 1 = 0$, or $p = 0.1$ narrowly approximates the truth. Accordingly, I write 0.1 in the quotient and, supposing $0.1 + q = p$, I substitute this fictitious value for it as before. There results $q^3 + 6.3q^2 + 11.23q + 0.061 = 0$. And since $11.23q + 0.061 = 0$ closely approaches the truth, in other words very nearly $q = -0.0054$ (by dividing 0.061 by 11.23, that is, until there are obtained as many figures as places which, excluding the bounding ones, lie between the first figures of this quotient and of the principal one – here, for instance, there are two between 2 and 0.005), I write – 0.0054 in the lower part of the quotient seeing that it is negative and then, supposing – 0.0054 + r equal to q, I substitute this value as previously. And in this way I extend the operation at pleasure after the manner of the diagram appended.

+ 2.10000000
– 0.00544852
2.09455148 [= y]

$2 + p = y$.	y^3	$+8$	$+12p$	$+6p^2$	$+p^3$
	$-2y$	-4	$-2p$		
	-5	-5			
	Total	-1	$+10p$	$+6p^2$	$+p^3$
$0.1 + q = p$.	$+p^3$	$+0.001$	$+0.03q$	$+0.3q^2$	$+q^3$
	$+6p^2$	$+0.06$	$+1.2$	$+6$	
	$+10p$	$+1$	$+10$		
	-1	-1			
	Total	0.061	$+11.23q$	$+6.3q^2$	$+q^3$
$-0.0054 + r = q$.	$+q^3$	$-0\cdot0000001\cancel{5}\cancel{7}\cancel{4}\cancel{6}\cancel{4}$	$+0.000\cancel{0}\cancel{8}\cancel{7}\cancel{4}\cancel{8}r$	$-\cancel{0}\cdot\cancel{0}\cancel{1}\cancel{6}\cancel{2}r^2$	$+\cancel{1}r^3$
	$+6.3q^2$	$+0\cdot0001837\cancel{0}\cancel{8}$	$-0\cdot068\cancel{0}\cancel{4}$	$+\cancel{6}\cdot\cancel{3}$	
	$+11.23q$	-0.060642	$+11.23$		
	$+0.061$	$+0.061$			
	Total	$+0.0005416$	$+11.162r$		
$-0.00004852 + s = r$.					

Near the end, however, (especially in equations of several dimensions) the work will be much shortened by this method. When you have decided how far you wish the root to be extracted, count off as many places from the first figure of the coefficient of the last term but one in the equations resulting on the right side of the diagram as there remain places to be filled up in the quotient, and neglect the decimals which follow after. But in the final term neglect the decimals after as many more places as there are decimal places filled up in the quotient, and in the last term but two neglect all after as many fewer. And so on, progressing arithmetically by that interval of places or, what is the same, cancelling everywhere as many figures as there are in the last but one term provided their lowest places be in arithmetical progression in accord with the series of terms, but alternatively these are to be understood to be filled up with zeros when the circumstances prove otherwise. Thus in the example now propounded, should I desire to complete the quotient to the eighth place of decimals only, while substituting 0.0054 + r for q (at which stage four decimal places in the quotient are entered and the same number remain to be filled in) I could have omitted figures in the five lower places: these I have on that account scored with a small oblique stroke – indeed, even though the first term r^3 had the coefficient 99999, I could have omitted it entirely. Consequently, when those figures are expunged, for the following operation the total comes to 0.0005416 + 11.162r, and this, upon performing division as far as the prescribed term, yields $-0\cdot00004852$ for r, so completing the quotient to the desired period. Finally I subtract the negative portion of the quotient from the positive one, and there arises 2.09455148 for the finished quotient