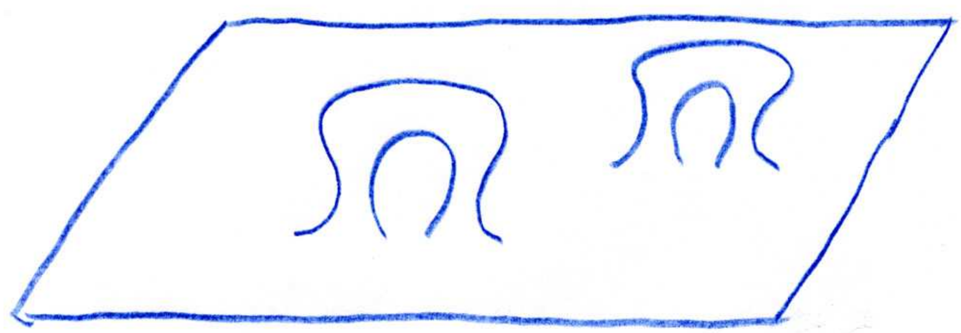
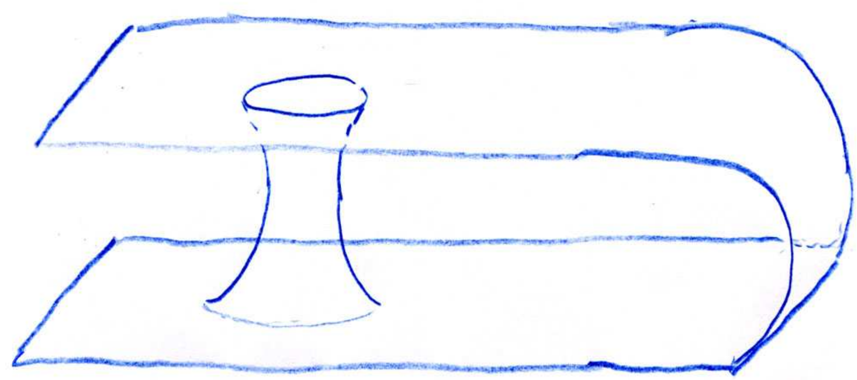
Topological Censorship

- Review the concept of Topological Censorship
- Review the Topological Censorship Theorem of Friedmann, Schleich and Witt
- Discuss extensions and applications (topology of black holes; AdS/CFT correspondence)

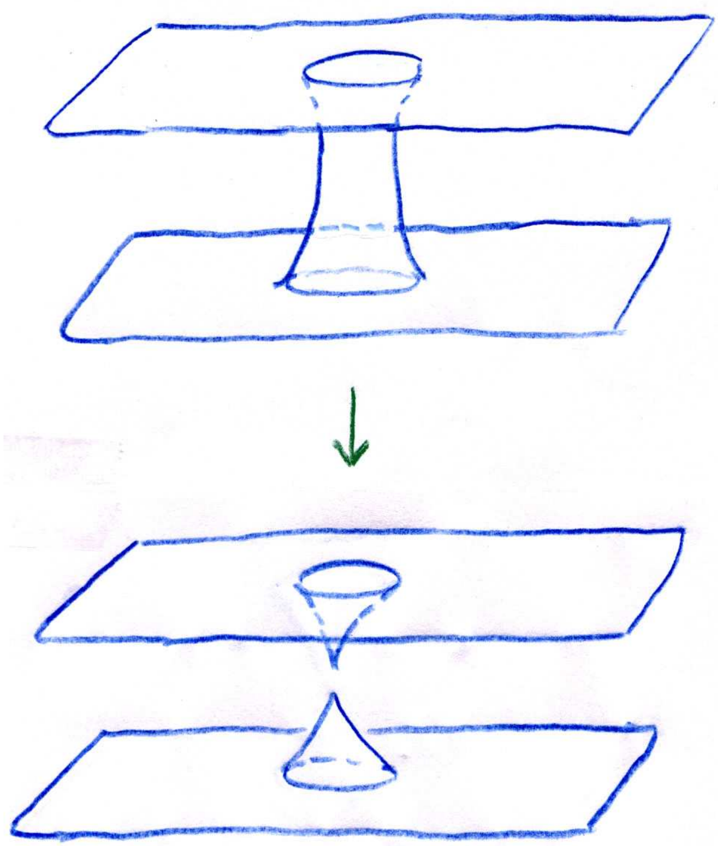
GR allows a priori the universe to have nontrivial topology - eg. wormholes, etc.





In principle this topology could be detected, e.g. wormholes could be traversed by observers or starlight, etc.

But from familiar examples, we know nontrivial topological structures tend to "pinch off" and form singularities



Gannon '75, Lee '76: Let M be spacetime obeying the null energy condition and let S be an AF Cauchy surface for M then

$$\Pi_1(S) \neq 0 \quad \Rightarrow \quad \text{spacetime is singular}$$

i.e., M is future null geodesically incomplete

These examples and results indicate that nontrivial topology tends to induce gravitational collapse.

According to the Cosmic Censorship Conjecture, the process of gravitational collapse leads to the formation of an event horizon which hides the singularities that develop from view.

This suggests a notion of Topological Censorship in which nontrivial topology becomes hidden behind the event horizon, and the region exterior to all black holes (and white holes) will have simple topology

(Gal. '93: Steady state spatial topology outside horizons is Euclidean – proved using minimal surface theory)

Principle of Topological Censorship (Friedman, Schleich and Witt '93) :

Every causal curve extending from past null infinity to future null infinity can be continuously deformed to a curve near infinity.

Roughly speaking, this says that an observer, whose trip begins and ends near infinity, and who thus remains outside all black holes, is unable to probe any nontrivial topological structures.

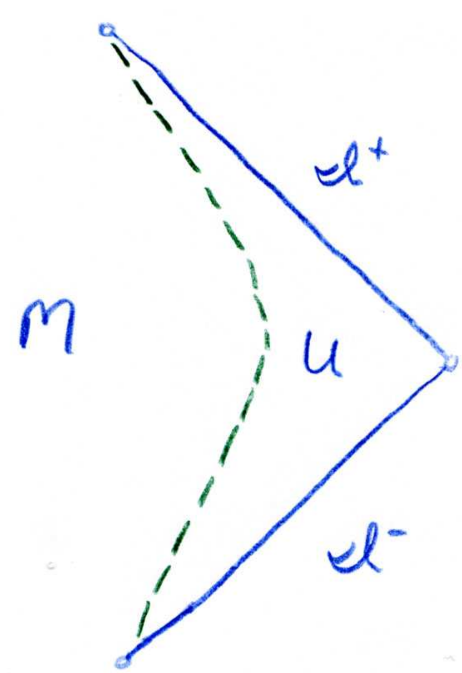
The topological censorship theorem of FSW applies to asymptotically flat spacetimes, i.e., spacetimes which "look like" Minkowski space near infinity.

(M, g_{ab}) is *asymptotically flat* if it admits a *conformal completion* $(\tilde{M}, \tilde{g}_{ab})$, a spacetime with boundary such that,

- $\tilde{M} = M \cup \mathcal{I}$, $\quad \mathcal{I} = \partial \tilde{M}$
- $\tilde{g}_{ab} = \Omega^2 g_{ab}$
- $\Omega = 0$ and $d\Omega \neq 0$ along $\mathcal{I}$

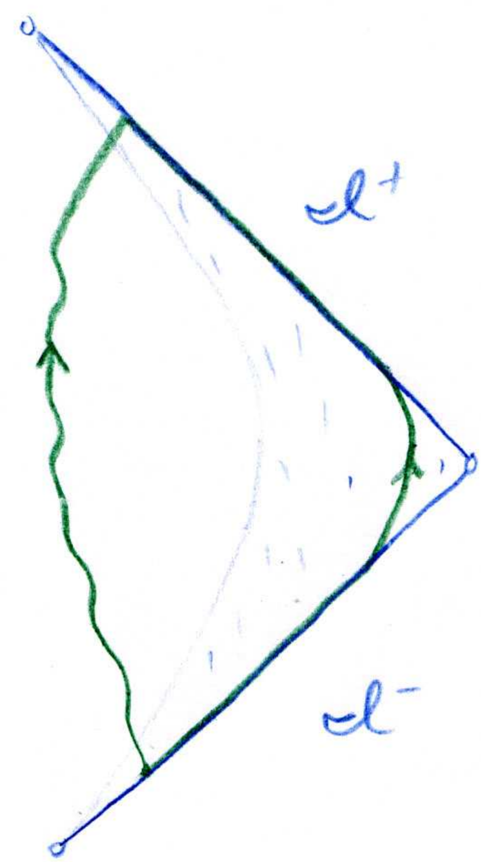
$$\mathcal{I} = \mathcal{I}^+ \cup \mathcal{I}^- \,,$$

$$\mathcal{I}^+ \approx \mathbb{R} \times S^2 \quad \text{and} \quad \mathcal{I}^- \approx \mathbb{R} \times S^2$$



$U =$ simply connected nbd of $\mathcal{I}$

Theorem (FSW '93) *Let (M, g_{ab}) be a globally hyperbolic, asymptotically flat spacetime which obeys the ANEC. Then every causal curve from $\mathcal{I}^-$ to $\mathcal{I}^+$ can be deformed to a curve lying in a simply connected neighborhood of $\mathcal{I}$.*



NEC: *null energy condition*

$$\text{Ric}(X, X) = R_{ij}\, X^i X^j \geq 0$$

for all null vectors X

ANEC: *average null energy condition*

$$\int_\eta \text{Ric}\,(\eta', \eta')\, d\lambda \geq 0$$

for all null geodesics starting near $\mathcal{J}^-$ and ending at $\mathcal{J}^+$.

If the Einstein equations hold,

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = 8\pi T_{ij}$$

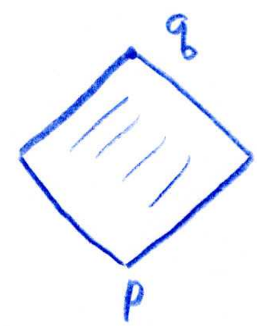
then

$$\text{Ric}\,(X, X) = 8\pi T_{ij} X^i X^j$$

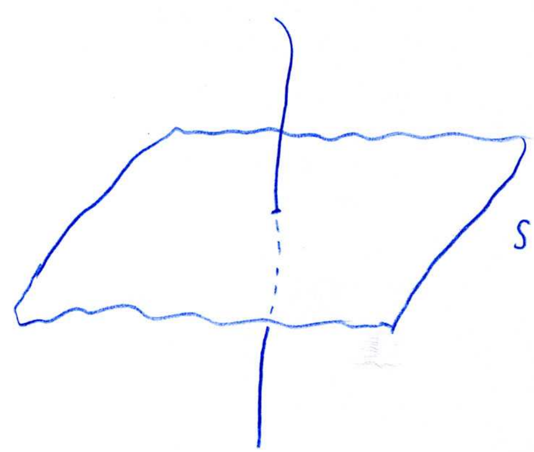
for all null vectors X. Hence, ANEC is insensitive to sign of the cosmological constant.

A spacetime M is globally hyperbolic provided

- M is strongly causal
- the sets $J^+(p) \cap J^-(q)$ are compact for all $p, q \in M$



Fact: M is globally hyperbolic iff it admits a Cauchy hypersurface S (a hypersurface which is met exactly once by every inextendible causal curve).



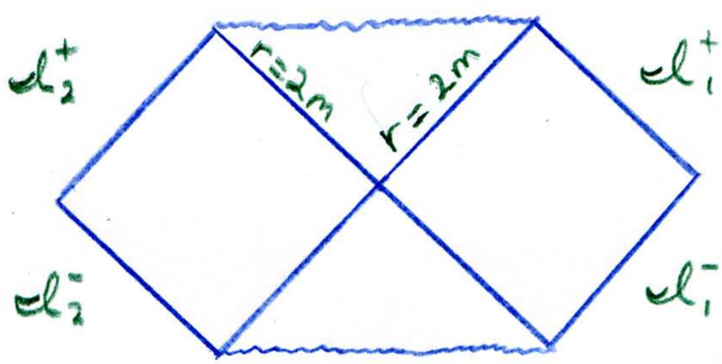
M globally hyperbolic $\Rightarrow M \approx \mathbb{R} \times S$.

The top cen theorem of FSW is closely related to a basic result concerning the *end structure* of spacetime.

A spacetime may have more than one null infinity,

$$\mathscr{I} = \cup_\alpha \mathscr{I}_\alpha = \cup_\alpha (\mathscr{I}_\alpha^+ \cup \mathscr{I}_\alpha^-)$$

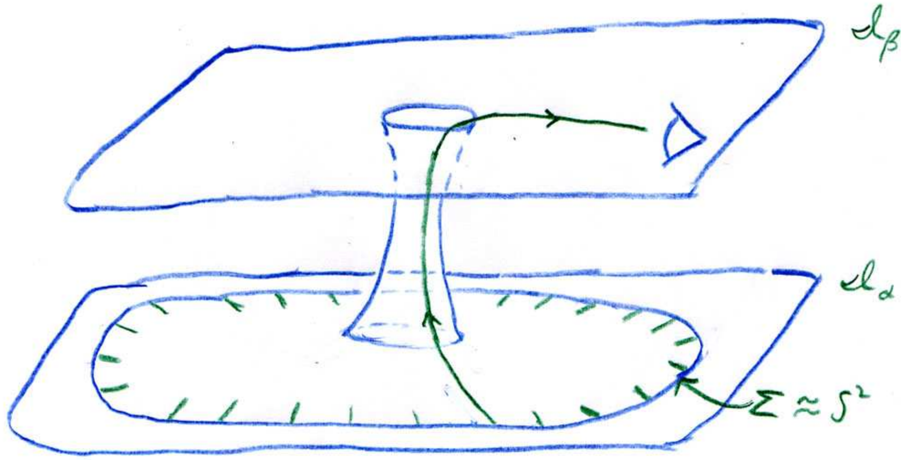
<u>Ex.</u> Maximally extended Schwarzschild spacetime.

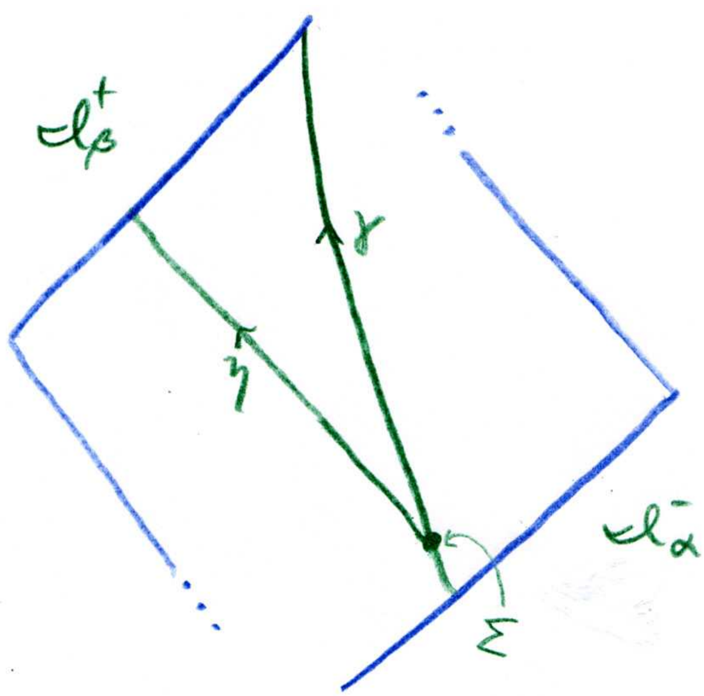


Proposition *Let M be a GH, AF spacetime which satisifies ANEC. Then no causal curve can extend from one null infinity to another, i.e.,*

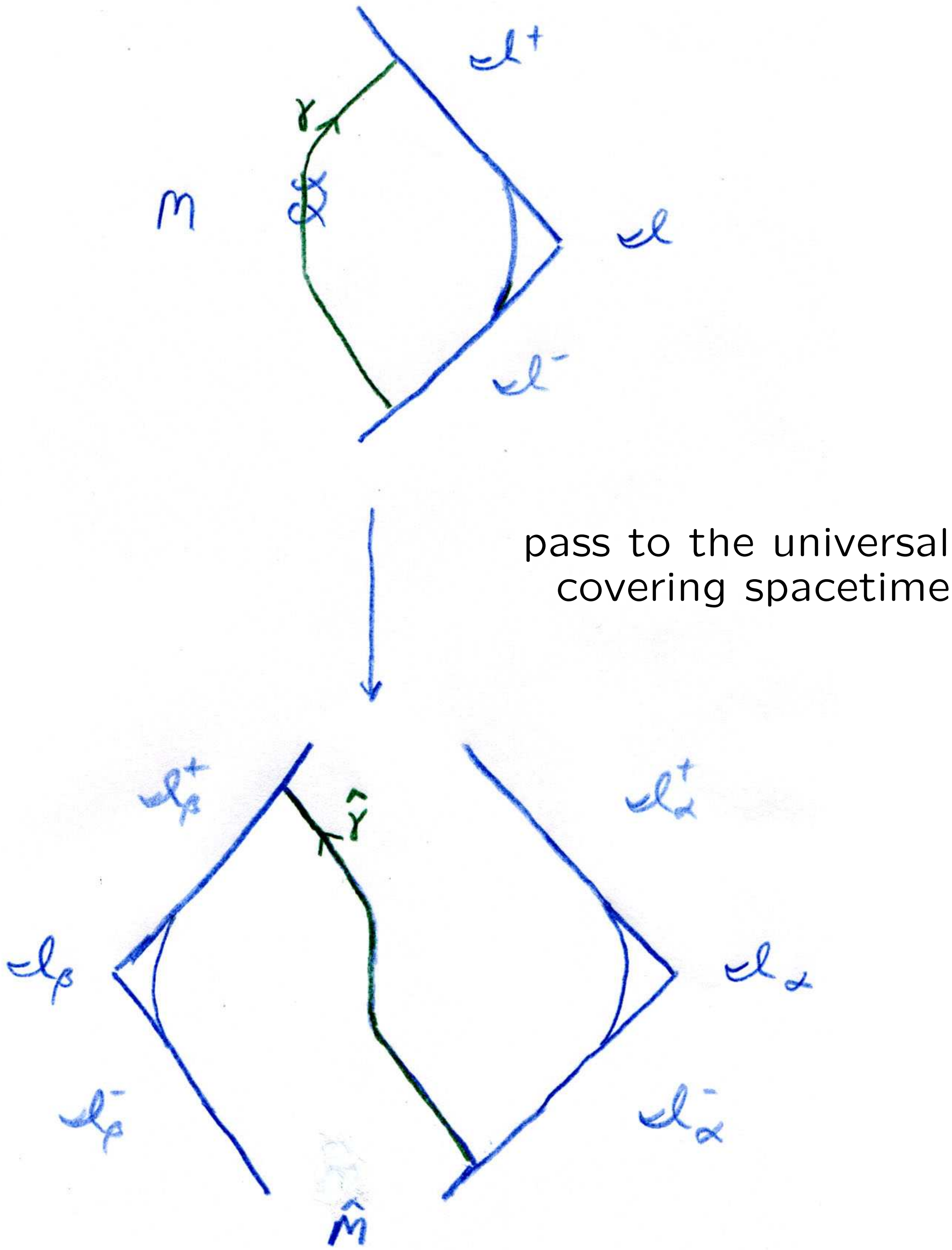
$$J^+(\mathscr{I}_\alpha^-) \cap J^-(\mathscr{I}_\beta^+) = \emptyset\,, \quad \alpha \neq \beta$$

The proposition essentially follows from the fact in black hole theory that outer trapped surfaces cannot be seen by observers near infinity.





Proof of the FSW Top Cen Theorem:



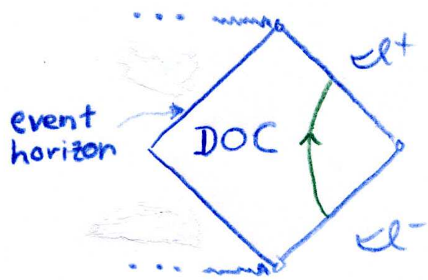
$\mathcal{I}_\alpha$ and $\mathcal{I}_\beta$ are in causal communication $\rightarrow\leftarrow$

Topological Censorship: the Strong Form

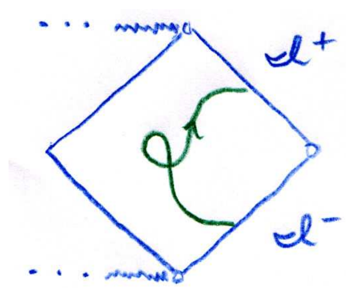
Top Cen is really a statement about the *domain of outer communications*,

$$\begin{aligned} DOC &= \text{region outside all black holes} \\ &\quad\ \text{and white holes} \\ &= J^-(\mathcal{I}^+) \cap J^+(\mathcal{I}^-) \end{aligned}$$

Top Cen: Observers in the DOC do not detect any nontrivial topology.



Top Cen (strong form): Observers in the DOC, even superluminal ones, do not detect any non-trivial topology.



Theorem (Gal '95) *Let* (M, g_{ab}) *be an asymptotically flat spacetime which obeys the ANEC, and suppose the DOC is globally hyperbolic. Then the DOC is simply connected,*

$$\Pi_1(DOC) = 0$$

Proof: Again uses the proposition, together with a slightly more involved covering space argument.

Browdy and Gal ('94, '95)

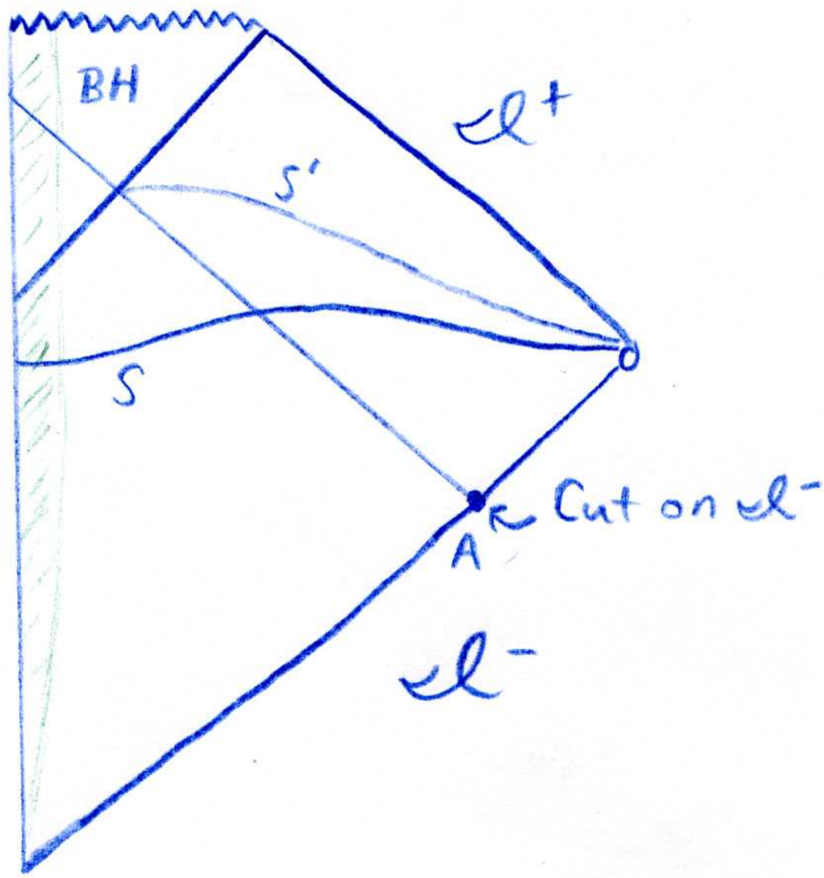
Jacobson and Venkataramani ('95)

Chruściel and Wald ('94)

Application to the topology of black holes

Top Cen provides a new proof of Hawking's black hole topology theorem, that black holes in AF spacetimes have spherical topology. Main advantages:

- Does not assume stationarity
- Uses the weaker null energy condition



S = Cauchy surface for DOC of M (misses horizon)

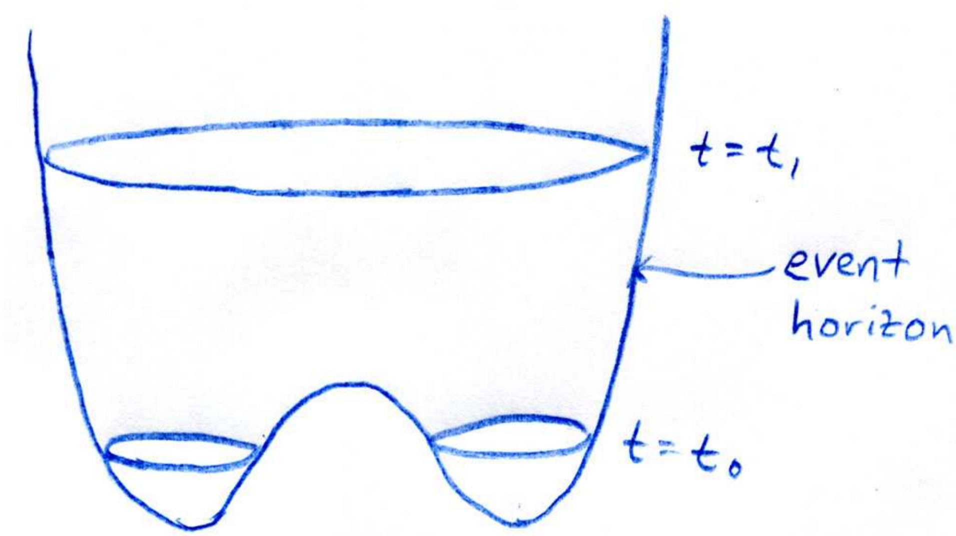
S' = Cauchy surface for DOC of $M' = I^+(A)$ (meets horizon)

Assume S' meets horizon in compact surface Σ. Top Cen $\Rightarrow$ DOC of M' simply connected $\Rightarrow$ S' simply connected $\Rightarrow \Sigma \approx S^2$.

Higher Genus Black Holes I:

Temporarily toroidal event horizons appearing in the numerical simulations of Shapiro, Teukolsky et al. ('94)

Toy model versions discussed in Shapiro, Teukolsky and Winicour ('95) and Gal, Schleich, Witt and Wooglar ('99)



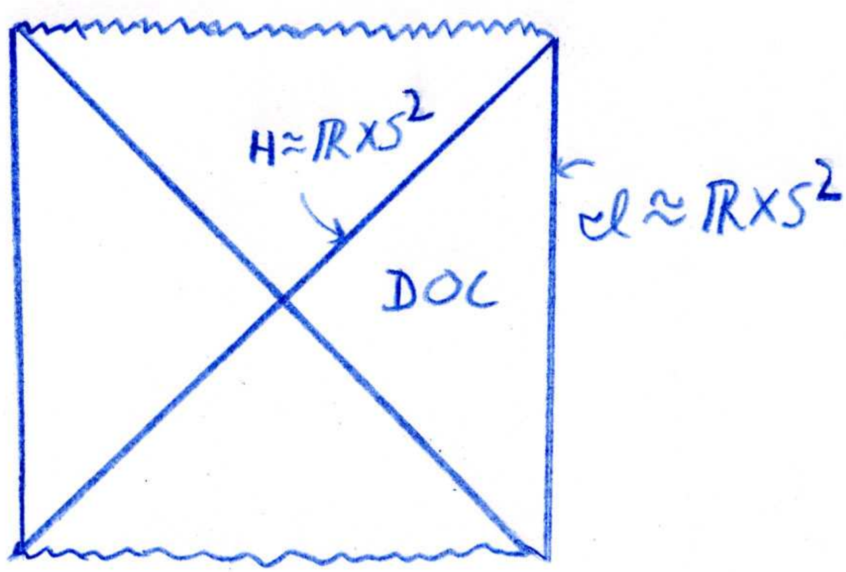
The DOC is simply connected, even though the spatial slice $t = t_0$ is not.

Top cen shows: *Eventually black hole topology is spherical.*

Higher Genus Black Holes II:

Black holes in asymptotically (locally) AdS space-times ($\Lambda < 0$)

Schwarzschild-AdS Spacetime (Kottler):

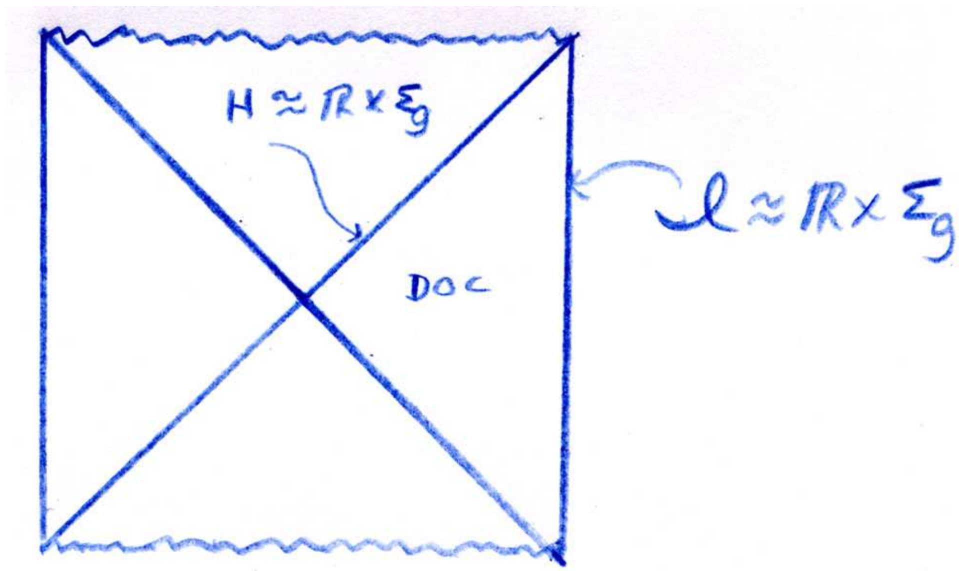


$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$V(r) = 1 - \frac{2m}{r} + \frac{|\Lambda|}{3}r^2$$

There exist higher genus counterparts to the SS-AdS spacetime.

(Bañados-Teitelboim-Zanelli, Lemos, Bengtsson et al., Brill-Louko-Peldán)



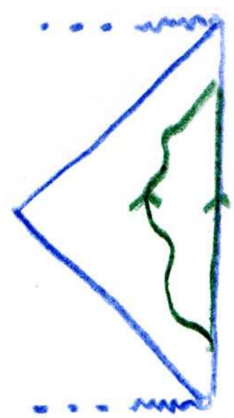
$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega_k^2$$

$$V(r) = k - \frac{2m}{r} + \frac{|\Lambda|}{3}r^2, \quad k = 0, \pm 1$$

Topological Censorship applies to the asymptotically locally AdS setting

Gal, Shleich, Witt, Woolgar ('99, '01)

Theorem *Let* (M, g_{ab}) *be an asymptotically locally AdS spacetime which obeys the ANEC, and suppose the DOC is globally hyperbolic (in the AdS sense). Then every curve (causal or otherwise) in the DOC with end points on* $\mathcal{I}$ *can be deformed to a curve lying in* $\mathcal{I}$*.*



$\Longrightarrow$ $\Pi_1(DOC)$ can be no more complicated than $\Pi_1(\mathcal{I})$ $\Longrightarrow$

$$g_0 \leq g_\infty \,,$$

where,

$$\begin{aligned} g_0 &= \text{genus of black hole} \\ g_\infty &= \text{genus of surface at infinity} \end{aligned}$$

Comments

- Main results on topological censorship hold in spacetime dimension ≥ 3. But these results do not determine black hole topology in higher dimensions.

- Topological censorship provides a spacetime analogue of the result of Witten and Yau ('99) concerning the "connectedness of the boundary" in the AdS/CFT correspondence (Gal, Schleich Witt, Woolgar '01)

  In the Euclidean setting (based on Witten's Euclidean formulation of the AdS/CFT correspondence) Witten and Yau prove under suitable circumstances that the conformal boundary at infinity is connected.

  In the Lorentzian setting, Top Cen shows that although conformal infinity may have multiple components, distinct components are not in causal contact.

$$\partial M = N_1 \cup N_2 \cup N_3$$