Introduction

In 1965 I first taught an undergraduate course in abstract algebra. It was fun to teach because the material was interesting and the class was outstanding. Five of those students later earned a Ph.D. in mathematics. Since then I have taught the course about a dozen times from various texts. Over the years I developed a set of lecture notes and in 1985 I had them typed so they could be used as a text. They now appear (in modified form) as the first five chapters of this book. Here were some of my motives at the time.

1) To have something as short and inexpensive as possible. In my experience, students like short books.

2) To avoid all innovation. To organize the material in the most simple-minded straightforward manner.

3) To order the material linearly. To the extent possible, each section should use the previous sections and be used in the following sections.

4) To omit as many topics as possible. This is a foundational course, not a topics course. If a topic is not used later, it should not be included. There are three good reasons for this. First, linear algebra has top priority. It is better to go forward and do more linear algebra than to stop and do more group and ring theory. Second, it is more important that students learn to organize and write proofs themselves than to cover more subject matter. Algebra is a perfect place to get started because there are many "easy" theorems to prove. There are many routine theorems stated here without proofs, and they may be considered as exercises for the students. Third, the material should be so fundamental that it be appropriate for students in the physical sciences and in computer science. Zillions of students take calculus and cookbook linear algebra, but few take abstract algebra courses. Something is wrong here, and one thing wrong is that the courses try to do too much group and ring theory and not enough matrix theory and linear algebra.

5) To offer an alternative for computer science majors to the standard discrete mathematics courses. Most of the material in the first four chapters of this text is covered in various discrete mathematics courses. Computer science majors might benefit by seeing this material organized from a purely mathematical viewpoint.

Over the years I used the five chapters that were typed as a base for my algebra courses, supplementing them as I saw fit. In 1996 I wrote a sixth chapter, giving enough material for a full first year graduate course. This chapter was written in the same "style" as the previous chapters, i.e., everything was right down to the nub. It hung together pretty well except for the last two sections on determinants and dual spaces. These were independent topics stuck on at the end. In the academic year 1997-98 I revised all six chapters and had them typed in LaTeX. This is the personal background of how this book came about.

It is difficult to do anything in life without help from friends, and many of my friends have contributed to this text. My sincere gratitude goes especially to Marilyn Gonzalez, Lourdes Robles, Marta Alpar, John Zweibel, Dmitry Gokhman, Brian Coomes, Huseyin Kocak, and Shulim Kaliman. To these and all who contributed, this book is fondly dedicated.

This book is a survey of abstract algebra with emphasis on linear algebra. It is intended for students in mathematics, computer science, and the physical sciences. The first three or four chapters can stand alone as a one semester course in abstract algebra. However they are structured to provide the background for the chapter on linear algebra. Chapter 2 is the most difficult part of the book because groups are written in additive and multiplicative notation, and the concept of coset is confusing at first. After Chapter 2 the book gets easier as you go along. Indeed, after the first four chapters, the linear algebra follows easily. Finishing the chapter on linear algebra gives a basic one year undergraduate course in abstract algebra. Chapter 6 continues the material to complete a first year graduate course. Classes with little background can do the first three chapters in the first semester, and chapters 4 and 5 in the second semester. More advanced classes can do four chapters the first semester and chapters 5 and 6 the second semester. As bare as the first four chapters are, you still have to truck right along to finish them in one semester.

The presentation is compact and tightly organized, but still somewhat informal. The proofs of many of the elementary theorems are omitted. These proofs are to be provided by the professor in class or assigned as homework exercises. There is a non-trivial theorem stated without proof in Chapter 4, namely the determinant of the product is the product of the determinants. For the proper flow of the course, this theorem should be assumed there without proof. The proof is contained in Chapter 6. The Jordan form should not be considered part of Chapter 5. It is stated there only as a reference for undergraduate courses. Finally, Chapter 6 is not written primarily for reference, but as an additional chapter for more advanced courses.

This text is written with the conviction that it is more effective to teach abstract and linear algebra as one coherent discipline rather than as two separate ones. Teaching abstract algebra and linear algebra as distinct courses results in a loss of synergy and a loss of momentum. Also with this text the professor does not extract the course from the text, but rather builds the course upon it. I am convinced it is easier to build a course from a base than to extract it from a big book. Because after you extract it, you still have to build it. The bare bones nature of this book adds to its flexibility, because you can build whatever course you want around it. Basic algebra is a subject of incredible elegance and utility, but it requires a lot of organization. This book is my attempt at that organization. Every effort has been extended to make the subject move rapidly and to make the flow from one topic to the next as seamless as possible. The student has limited time during the semester for serious study, and this time should be allocated with care. The professor picks which topics to assign for serious study and which ones to "wave arms at". The goal is to stay focused and go forward, because mathematics is learned in hindsight. I would have made the book shorter, but I did not have any more time.

When using this text, the student already has the outline of the next lecture, and each assignment should include the study of the next few pages. Study forward, not just back. A few minutes of preparation does wonders to leverage classroom learning, and this book is intended to be used in that manner. The purpose of class is to learn, not to do transcription work. When students come to class cold and spend the period taking notes, they participate little and learn little. This leads to a dead class and also to the bad psychology of "O K, I am here, so teach me the subject." Mathematics is not taught, it is learned, and many students never learn how to learn. Professors should give more direction in that regard.

Unfortunately mathematics is a difficult and heavy subject. The style and approach of this book is to make it a little lighter. This book works best when viewed lightly and read as a story. I hope the students and professors who try it, enjoy it.

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