Practice problems for Exam 2

To practice for the second exam, you should go over the homework problems, especially from Homeworks 5–8. Here are some additional practice problems.

1. Let $A_1, A_2, \ldots$ be a sequence of subspaces of a topological space X. Suppose each A_i is connected and $A_i \cap A_{i+1} \neq \varnothing$ for all $i \in \mathbb{Z}_+$. Show that $\bigcup_{i \in \mathbb{Z}_+} A_i$ is connected.

2. Show that in the finite complement topology on $\mathbb{R}$, every subspace is compact.

3. Let $f : X \to Y$ be a continuous bijective function from a compact topological space X to a Hausdorff topological space Y. Prove that f is a homeomorphism.

4. Prove that every non-constant path in $\mathbb{R}^2$ (i.e. continuous function from closed interval to $\mathbb{R}^2$) must contain a point with at least one of whose coordinates is rational.

5. Let X be a topological space with an equivalence relation $\sim$. Show by example that even if X Hausdorff, the quotient space $X/\sim$ need not be Hausdorff. Similarly, show that even if $X/\sim$ is Hausdorff, X need not be.