Homework 9

Due: 11/20/2019, in class

MTH 311 Sections C and F

Fall 2019

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

1. Consider a non-linear system which models populations $x(t)$ and $y(t)$:

$$\begin{cases} x' = -4x + 2xy \\ y' = -3y + xy \end{cases} \tag{1}$$

Note the signs of the terms. The -4 indicates that the population $x(t)$ decays exponentially in the absence of $y(t)$; the -3 indicates the same for $y(t)$, in the absence of the population $x(t)$. The coefficients in front of both xy terms are positive, which means that each population benefits from the other.

   (a) Find the equilibrium points of the system.

   (b) For each equilibrium point (a, b), linearize the system at (a, b), find the associated eigenvalues and eigenvectors, and draw the linear phase portait.

   (c) Draw a phase portrait for the total non-linear system (1).

   (d) Indicate on your phase portrait for which initial $(x(0), y(0))$ the two populations x and y survive in the long run.

System (1) models populations in **cooperation**. If the signs of the xy terms are instead negative, the populations are in **competition**.

2. Consider a non-linear system which models populations $x(t)$ and $y(t)$:

$$\begin{cases} x' = 3x - x^2 - \frac{1}{4}xy \\ y' = -2y + xy \end{cases} \tag{2}$$

This is an example of a (modified) predator-prey model, as discussed in class.

   (a) Which population is predator, and which is prey?

   (b) Find the equilibrium points of the system.

   (c) For each equilibrium point (a, b), linearize the system at (a, b), find the associated eigenvalues and eigenvectors, and draw the linear phase portait.

   (d) Draw a phase portrait for the total non-linear system (2).