Homework 9 solutions

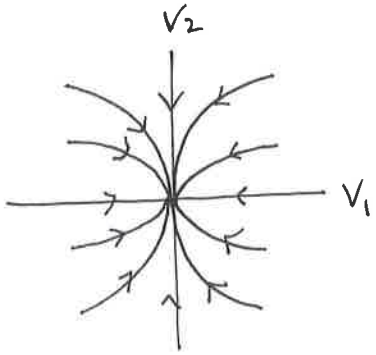
1. $\begin{cases} x' = -4x + 2xy \\ y' = -3y + xy \end{cases}$

(a) Equilibria: $(0,0)$, $(3,2)$

$$A(x,y) = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} = \begin{pmatrix} -4+2y & 2x \\ y & -3+x \end{pmatrix}$$

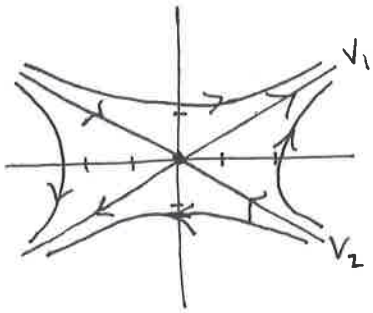
(b) $\underline{(0,0)}$: $A(0,0) = \begin{pmatrix} -4 & 0 \\ 0 & -3 \end{pmatrix}$ eigenvalues: $\lambda_1 = -4$, $\lambda_2 = -3$

eigenvectors: $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

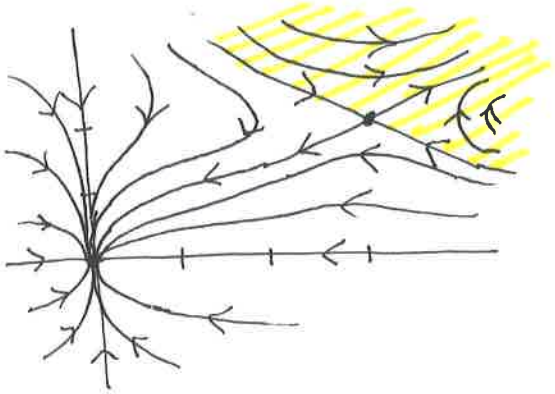


$\underline{(3,2)}$: $A(3,2) = \begin{pmatrix} 0 & 6 \\ 2 & 0 \end{pmatrix}$ eigenvalues: $\lambda_1 = 2\sqrt{3}$, $\lambda_2 = -2\sqrt{3}$

eigenvectors: $v_1 = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$



(c) total phase portrait:



(d) Points $(x(0), y(0))$ of initial populations for which x and y survive in the long run lie in the shaded region.

2. $\begin{cases} x' = 3x - x^2 - \frac{1}{4}xy \\ y' = -2y + xy \end{cases}$

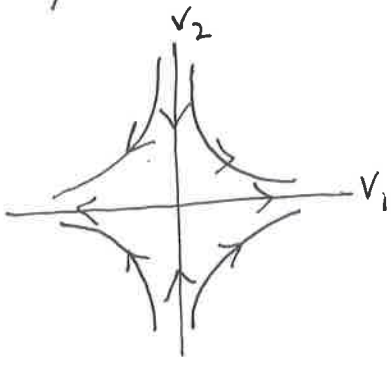
(a) x = prey
y = predator

(b) Equilibria: $(0,0)$, $(2,4)$, $(3,0)$

$$A(x,y) = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} = \begin{pmatrix} 3-2x-\frac{1}{4}y & -\frac{1}{4}x \\ y & -2+x \end{pmatrix}$$

(c) $\underline{(0,0)}$: $A(0,0) = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$ eigenvalues: $\lambda_1 = 3$, $\lambda_2 = -2$

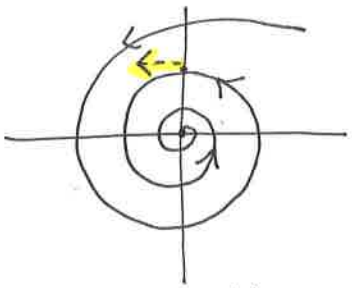
eigenvectors: $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



(2,4): $A(2,4) = \begin{pmatrix} -2 & -\frac{1}{2} \\ 4 & 0 \end{pmatrix}$ eigenvalues: $-1 \pm i$.

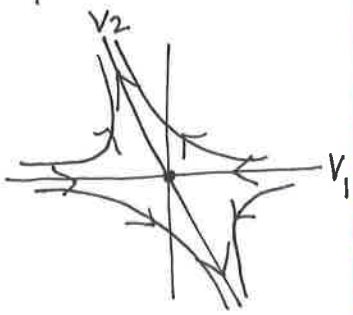
tangency $\vec{x}'$ at $\vec{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$:

$\begin{pmatrix} -2 & -\frac{1}{2} \\ 4 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}$



(3,0): $A(3,0) = \begin{pmatrix} -3 & -\frac{3}{4} \\ 0 & 1 \end{pmatrix}$ eigenvalues: $\lambda_1 = -3$, $\lambda_2 = 1$

eigenvectors: $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 3 \\ -16 \end{pmatrix}$



(d) total phase portrait: