Homework 8 solutions

1. (a) $AB + I = \begin{pmatrix} -9 & 10 & 2 \\ 0 & 6 & 3 \\ 14 & -1 & 6 \end{pmatrix}$ (b) $BC - 2B = \begin{pmatrix} 9 & 1 & 3 \\ -7 & 7 & 3 \end{pmatrix}$

(c) $CE + 3E = \begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix}$

2. (a) $A = \begin{pmatrix} 6 & -7 \\ 1 & -2 \end{pmatrix}$ eigenvalues: $\lambda_1 = -1$, $\lambda_2 = 5$
eigenvectors: $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

Any non-zero multiple of v_1 (resp. v_2) is an eigenvector for λ_1 (resp. λ_2) (Similar remark for below.)

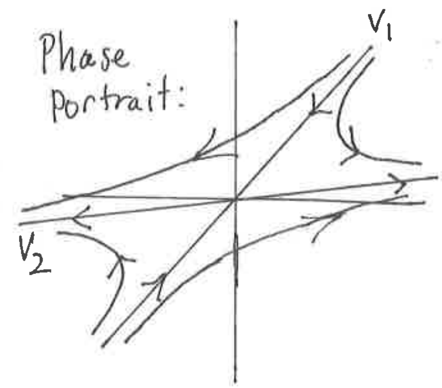
(b) $A = \begin{pmatrix} 4 & 1 & 4 \\ 1 & 7 & 1 \\ 4 & 1 & 4 \end{pmatrix}$ eigenvalues: $\lambda_1 = 0$, $\lambda_2 = 6$, $\lambda_3 = 9$
Eigenvectors: $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(c) $A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ eigenvalues: $\lambda_1 = 2$ (mult. 2)
eigenvectors: $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, and nonzero multiples.

3. $\vec{x}\,' = A\vec{x}$, $A = \begin{pmatrix} 6 & -7 \\ 1 & -2 \end{pmatrix}$ General solution:

$$\vec{x}(t) = c_1 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{5t}\begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

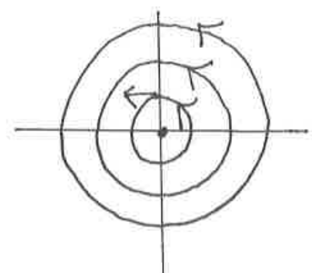
Phase Portrait:



4. $\vec{x}\,' = A\vec{x}$, $A = \begin{pmatrix} 0 & -1 \\ 4 & 0 \end{pmatrix}$

General solution: $\vec{x}(t) = c_1 \begin{pmatrix} \cos(2t) \\ 2\sin(2t) \end{pmatrix} + c_2 \begin{pmatrix} \sin(2t) \\ -2\cos(2t) \end{pmatrix}$

Phase portrait:



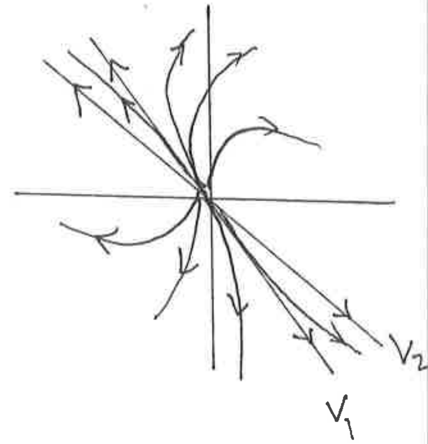
Tangency at $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$: $\begin{pmatrix} 0 & -1 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

5. $\vec{x}' = A\vec{x}$, $A = \begin{pmatrix} 9 & 5 \\ -6 & -2 \end{pmatrix}$

General solution:

$$\vec{x}(t) = c_1 e^{3t}\begin{pmatrix} 5 \\ -6 \end{pmatrix} + c_2 e^{4t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

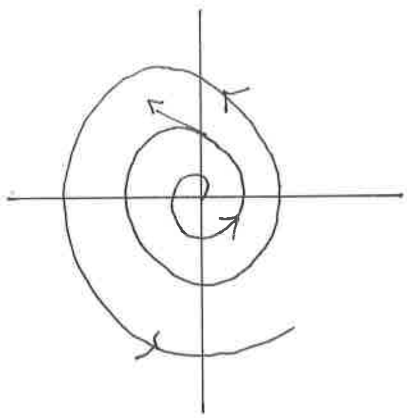
Phase portrait:



6. $\vec{x}' = A\vec{x}$, $A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

General solution: $\vec{x}(t) = c_1 e^{t}\begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} + c_2 e^{t}\begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

Phase portrait:



Tangency at $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$:

$$\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$