

Homework 6

Due: 10/18/2019, in class

MTH 311 Sections C and F

Fall 2019

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

1. A body with mass 150 kg is attached to the end of a spring that is stretched 10 cm by a force of 5 N. At $t = 0$ the body is pushed 1 m to the left (in the usual picture) and set in motion with an initial velocity of 4 m/s.
 - (a) Find $x(t)$ in the form $C \cos(\omega_0 t - \alpha)$ and graph it.
 - (b) What are the amplitude and period of motion of the body?
2. Suppose a body with mass 2 kg is attached to a spring that is stretched 2 m by a force of 4 N, but now also a shock absorber is attached to the other end of the body. At $t = 0$ the body is pulled 2 m to the right (in the usual picture) and set in motion with an initial velocity of 1 m/s.
 - (a) Suppose the damping constant is $b = 6$. Find $x(t)$. If your answer involves sines and cosines, write it in the form $x(t) = Ce^{-pt} \cos(\omega_1 t - \alpha)$.
 - (b) Do the same as in (a) but for $b = 4$.
 - (c) Do the same as in (a) but for $b = 2$.
3. For each of the following equations find a particular solution $y_p(t)$.
 - (a) $y'' + 4y = e^{5t}$
 - (b) $4y'' + 4y' + y = 3te^t$
 - (c) $y'' + 4y' + 2y = t^2$
 - (d) $y'' + 9y = \cos(3t) + 4\sin(3t)$
4. For each of the following equations find the general solution.
 - (a) $y'' - 2y' + 2y = e^t \sin(t)$
 - (b) $y^{(4)} - 4y'' + 4y = e^t - te^{2t}$
5. Suppose a body with mass 1 kg is attached to a spring that is stretched 2 m by a force of 2 N. There is also an external force of $\cos(t)$ Newtons acting on the spring. At $t = 0$ the body is 1 m to the left (in the usual picture) and has an initial velocity of 1 m/s. Find $x(t)$.