Homework 5

MTH 311 Sections C and F

Due: 10/11/2019, in class

Fall 2019

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

1. Show that $y_1(t) = \sqrt{t}$ and the constant function $y_2(t) = 1$ are solutions of $yy'' + (y')^2 = 0$, but that the sum $y_1 + y_2$ is not a solution. (This shows that the superposition principle does not hold for nonlinear equations.)

2. Determine whether each pair of functions y_1, y_2 are linearly independent.
   (a) $y_1(t) = e^t \sin(t)$, $y_2(t) = e^t \cos(t)$ (Try the Wronskian on this one!)
   (b) $y_1(t) = \sin^2(t)$, $y_2(t) = 1 - 2\cos(2t)$
   (c) $y_1(t) = t^3$, $y_2(t) = t^2|t|$

3. Find the general solution to each of the following differential equations.
   (a) $y'' - 3y' + 2y = 0$
   (b) $y'' - 10y' = 0$
   (c) $y'' + y' - y = 0$
   (d) $y'' + 2y' + y = 0$

4. Find the solution to $y'' + 5y' + 6y = 0$ satisfying $y(0) = 0$ and $y'(0) = 1$.

5. Find the general solution to each of the following differential equations.
   (a) $3y^{(4)} + 4y^{(3)} = 0$
   (b) $y^{(4)} - 3y^{(3)} + 3y'' - y' = 0$

6. Find the general solution to each of the following differential equations.
   (a) $y'' - 2y' + 2y = 0$
   (b) $y'' + y' + y = 0$

7. Each of the below is a general solution $y(t)$ of a homogeneous 2^{nd} order ODE $y'' + py' + qy = 0$ with constant coefficients. Find such an equation.
   (a) $y(t) = c_1 e^{-5t} + c_2 t e^{-5t}$
   (b) $y(t) = e^t \left(c_1 e^{t\sqrt{3}} + c_2 e^{-t\sqrt{3}}\right)$
   (c) $y(t) = c_1 e^t \cos(2t) + c_2 e^t \sin(2t)$