Homework 4

MTH 311 Sections C and F

Due: 09/20/2019, in class

Fall 2019

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

1. Consider the initial value problem $y' = -8ty$, $y(0) = 2$.
   (a) Apply Euler's method to estimate $y(1)$. Use step sizes $\Delta t = 1/4$.
   (b) Find an explicit solution, and compare your estimate to the actual $y(1)$.
2. Consider a population $P(t)$ satisfying the logistic equation $dP/dt = aP - bP^2$, where $B = aP$ is the time rate at which births occur, and $D = bP^2$ is the time rate at which deaths occur. Show the limiting population as $t \to \infty$ is a/b.
3. Consider a population of ibis satisfying the logistic equation as in the previous problem. Suppose there is an initial population of 30 ibis, and at $t = 0$ there are 6 births per month and 2 deaths per month. How many months will it take the ibis to reach 2/3 of their limiting population?
4. Consider the differential equation $dy/dt = (y - 3)^2$.
   (a) Find the equilibria (i.e. constant solutions).
   (b) By analyzing the sign of dy/dt around the equilibria, determine whether each equilibrium solution is stable, unstable, or neither.
   (c) Find an explicit general solution to the equation.
   (d) Graph the equilibrium solutions, as well as some non-constant solution curves, verifying visually the stability properties determined in (b).
5. Repeat parts (a) through (d) of Problem 4 for $y' = -18 + 11y - y^2$.
6. The differential equation $dP/dt = \frac{1}{2}P(3 - P) - H$ models a logistic population with harvesting at rate H. Determine the dependence of the number of equilibria on the parameter H, and draw a bifurcation diagram.