Homework 3

MTH 311 Sections C and F

Due: 09/13/2019, in class

Fall 2019

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

For slope fields: The amount of line segments you decide to include is up to you, as long as a pattern is visible from what you have drawn. (A few dozen is plenty.)

1. Consider the differential equation $y' = y - t$.
   (a) Construct a slope field for this equation.
   (b) Find the general solution to this differential equation.
   (c) There is exactly one solution that is given by a straight line. Write the equation for this line and draw it on the slope field.
2. Consider the differential equation $y' = t/y$.
   (a) Construct a slope field for this equation, omitting the origin $(0, 0)$.
   (b) Find the general solution to this differential equation.
   (c) There are two solutions that are given by lines. Write the equations for them, and draw them on the slope field.
3. Consider the differential equation $x\frac{dy}{dx} - y = 0$.
   (a) Construct a slope field for this equation, omitting the origin $(0, 0)$.
   (b) Find the general solution to this differential equation.
   (c) How many of the solutions pass through the origin?
4. Let c be a constant. Define a continuous function $y_c(t)$ piecewise by

$$y_c(t) = \begin{cases} 0 & \text{if } t \leqslant c \\ (t-c)^2 & \text{if } t > c \end{cases}$$

   (a) Verify that this function is a solution to $y' = 2\sqrt{y}$.
   (b) Graph the functions $y_c(t)$ for $c = -1, 0, 1, 2$.
   (c) How many solutions are there to $y' = 2\sqrt{y}$ which satisfy $y(0) = 0$?