

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

1. Solve the following initial value problem:

$$ty' + y = t + 1, \quad y(1) = 0$$

2. Solve the following initial value problem:

$$\frac{dy}{dx} + e^x y = e^x, \quad y(0) = 0$$

3. Solve the following initial value problem:

$$\frac{dy}{dt} + (\tan t)y = \sec t, \quad y(\pi/4) = 0$$

4. Find a general solution to the differential equation:

$$xy' + (2x - 3)y = 4x^4$$

5. A tank contains 500 liters (L) of a solution consisting of 80 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 10 L/sec, and the mixture is pumped out at the same rate. How long will it be until only 10 kg of salt remains in the tank?
6. In class, we focused on the method of “variation of parameters” to solve a given 1st order linear differential equation. However, we also derived the following general solution to the linear equation $y' + P(t)y = Q(t)$:

$$y(t) = e^{-\int P(t)dt} \left[\int Q(t)e^{\int P(t)dt} dt + C \right]$$

Here C is a constant. Verify directly that this expression in fact gives a solution.