Homework 10 w/solutions

MTH 311 Sections C and F

Due: 12/2/2019, in class

Fall 2019

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

1. Compute the Laplace transforms of the following from the definition of $\mathscr{L}\{f(t)\}$.

   (a) $f(t) = t - 2e^{2t}$ $\quad \mathscr{L}\{f(t)\} = 1/s^2 - 2/(s-2)$

   (b) $f(t) = (1+t)^3$ $\quad \mathscr{L}\{f(t)\} = 6/s^4 + 6/s^3 + 3/s^2 + 1/s$

2. Using the table in the textbook, find the inverse Laplace transforms of the following functions.

   (a) $F(s) = \dfrac{3}{s^5}$ $\quad \mathscr{L}^{-1}\{F(s)\} = t^4/8$

   (b) $F(s) = \dfrac{2}{3-s}$ $\quad \mathscr{L}^{-1}\{F(s)\} = -2e^{3t}$

   (c) $F(s) = \dfrac{10s-3}{25+s^2}$ $\quad \mathscr{L}^{-1}\{F(s)\} = 10\cos(5t) - \frac{3}{5}\sin(5t)$

3. Solve $y'' + 9y = 1$, $y(0) = y'(0) = 0$ using the Laplace transform method.
   $y(t) = \dfrac{2}{9}\sin^2\left(\dfrac{3t}{2}\right)$

4. Recall the translation property $\mathscr{L}\{e^{at}f(t)\} = F(s-a)$ where $F(s) = \mathscr{L}\{f(t)\}$. Use this to find the Laplace transform of $f(t) = e^{-2t}\sin(3\pi t)$.
   $\mathscr{L}\{f(t)\} = \dfrac{3\pi}{(s+2)^2 + 9\pi^2}$

5. Use the translation property to find the inverse Laplace transforms of:

   (a) $F(s) = \dfrac{s-1}{(s+1)^3}$ $\quad \mathscr{L}^{-1}\{F(s)\} = e^{-t}t(1-t)$

   (b) $F(s) = \dfrac{s+2}{s^2+4s+5}$ $\quad \mathscr{L}^{-1}\{F(s)\} = e^{-2t}\cos(t)$

6. Solve $y'' + 4y' + 13y = te^{-t}$, $y(0) = 0$, $y'(0) = 2$ using Laplace transforms.
   $y(t) = \frac{1}{50}e^{-2t}(e^t(-1+5t) + \cos(3t) + 32\sin(3t))$

7. Consider a spring system with $k = 4$, $m = 1$ and no shock absorber. The spring begins in equilibrium, at rest. At $t = 0$ an external force $f(t)$ is applied, which has $f(t) = 1$ for $0 \leqslant t < \pi$ and $f(t) = 0$ for $t \geqslant \pi$. Find $x(t)$.
   $x(t) = \dfrac{1}{2}(1 - u_\pi(t))\sin^2(t)$

8. Solve $y'' + 2y' + 2y = 2\delta(t-\pi)$, $y(0) = y'(0) = 0$ using Laplace transforms.
   $y(t) = -2e^{\pi - t}u_\pi(t)\sin(t)$