Homework 1

MTH 311 Sections C and F

Due: 08/30/2019, in class

Fall 2019

Please write legibly and show all work. If the answer to a problem is written down correctly, but certain steps of solving it are not shown, points might be taken off.

1. Verify that $y(t) = e^{t-t^2/2}$ is a solution to the differential equation
$$y'' + 2ty' + t^2 y = 0$$
2. Find a solution to the following, and satisfying the initial condition $y(0) = 0$:
$$\frac{dy}{dx} = \frac{x}{\sqrt{2-x^2}}$$
What is the domain of your solution?
3. Find a solution to $y'(t) = te^{-t}$ satisfying the condition $y(1) = 1$.
4. A car is driving 100 mph on the turnpike. The driver suddenly notices that there is standstill traffic ahead, at a distance of 7/10 of a mile, and slams on the brakes, providing a (constant) deceleration of 7500 miles/hour^2. Does the car avoid a crash, and if so, by how much of a distance?
5. Find general solutions (i.e. all the solutions you can possibly find using the methods discussed in class) to $y' + ty^2 = 0$.
6. Similarly, find general solutions to $\dfrac{dy}{dx} = 3\sqrt{xy + x}$.
7. Find a particular solution to $y\dfrac{dy}{dt} + \dfrac{t}{\sqrt{t^2-4}} = 0$ with $y(2\sqrt{2}) = 2$.
8. Find a particular solution to $y' = e^{y-t}$ with $y(0) = -\ln(2)$.
9. The city of Miami had a population of 362,220 in the year 2000, and a population of 463,347 in 2017. If the city's population continues to grow exponentially at a constant rate, what will the population be in 2050?
10. In a certain culture of bacteria, the number of bacteria quadrupled in 10 minutes. How long did it take for the bacteria to double?
11. The half-life of cobalt is 5.27 years. A nuclear disaster has left the level of cobalt radiation in a certain city at 120 times the level acceptable for human habitation. How long will it be until people can return to live in the city?
12. A large vat of coffee is brewed at 120° F. You're outside in Miami during the summer, so you throw an ice cube in the coffee, unaware that the temperature of the coffee will essentially not change because of one ice cube. The ice is 10° F. In 1 minute, the ice is 20° F. How much longer until the ice melts?