Practice Midterm 2

with solutions in blue

MTH 311 Sections C and F

Fall 2019

This is intended to be similar (but different!) to the midterm you will take in class. Make sure you also study the homework problems (from HW 5, 6, 7).

1. Find the general solution to the following differential equations.

   (a) $y'' - 8y' + 15y = 0$
   $y(t) = c_1 e^{5t} + c_2 e^{3t}$

   (b) $y'' - 2y' + 2y = 0$
   $y(t) = c_1 e^t \cos(t) + c_2 e^t \sin(t)$

   (c) $(D+2)^3(D-7)y = 0$ where $D = \frac{d}{dt}$
   $y(t) = c_1 e^{-2t} + c_2 t e^{-2t} + c_3 t^2 e^{-2t} + c_4 e^{7t}$

2. For the following equations, make a simplified guess for form of the particular solution. Do not solve for the coefficients.

   (a) $y'' + 4y = 5t\sin(2t) - t$
   $y_p(t) = c_1 + c_2 t + c_3 t\cos(t) + c_4 t\sin(t) + c_5 t^2\cos(t) + c_6 t^2\sin(t)$

   (b) $y^{(3)} + 2y'' + y' = -2e^{-t}\cos(t) + 3$
   $y_p(t) = c_1 e^{-t}\cos(t) + c_2 e^{-t}\sin(t) + c_3 t$

   (c) $y^{(4)} + 2y'' + y = \cos(t)$
   $y_p(t) = c_1 t^2\cos(t) + c_2 t^2\sin(t)$

3. Solve $y'' - y = e^t(2\cos(t) - \sin(t))$ with initial conditions $y(0) = y'(0) = 0$.
   $y(t) = e^{-t}/2 - e^t/2 + e^t\sin(t)$

4. Use variation of parameters to find a particular solution to $y'' + 2y' + y = 15e^{-t}\sqrt{t}$.
   $y_p(t) = 4e^{-t}t^{5/2}$

5. Consider a spring system with mass $m = 2$, spring constant $k = 20$, a shock absorber with damping constant $b = 4$, and an external force $f(t) = 3\cos(t)$.

   (a) Set up the ODE for the motion of the mass $x(t)$.
   $x'' + 2x' + 10x = 3\cos(t)/2$

   (b) Write the general solution, without solving for the coefficients of x_p.
   $x(t) = c_1 e^{-t}\cos(3t) + c_2 e^{-t}\sin(3t) + x_p(t)$ where $x_p(t) = \widetilde{c_1}\cos(t) + \widetilde{c_2}\sin(t)$

6. Find the solution to $y^{(3)} - 2y'' + 4y' - 8y = 0$ with $y(0) = 0$, $y'(0) = 4$, $y''(0) = 16$.
   $y(t) = 2e^{2t} - 2\cos(2t)$