# Practice Midterm 1

MTH 311 Sections C and F
Fall 2019

This is intended to be similar (but, of course, different...) to the midterm you will take in class. Make sure you also study the homework problems.

1. Consider the differential equation $y' = y^2 - 4$.
   (a) Find the equilibrium solutions, and draw the phase diagram.
   (b) In the $(t, y)$-plane, roughly sketch the three solutions with initial conditions $y(0) = 1.9$, $y(0) = 2.1$ and $y(0) = -4$.
2. Suppose a tank starts with 30 gallons of clean water. There are two pipes leading into the tank: one has salt water with 0.5 lb/gal of salt entering at a rate of 2 gal/min, while the other has 1 gallon of clean water entering at a rate of 1 gal/min. Finally, there is a pipe that lets salt water leave the tank at 3 gal/min. Find an expression for $S(t)$, the amount of salt at time $t$.
3. Solve the differential equation $y' = 1 + te^{-y}$ using the substitution $u = e^y$.
4. Consider $y' = y^2/t$ with initial condition $y(1) = 1$. Use Euler's method with $\Delta t = 1$ to estimate $y(4)$.
5. Consider $y' = y + t^2$.
   (a) Draw a slope field for this equation.
   (b) Do the solution curves ever intersect? Explain.
6. Find general solutions to the following equations.
   (a) $ty' - y = t^2 \cos(t)$
   (b) $y' + ty = t$
7. An apple pie was taken out of the oven and placed on a porch 25% as hot as the oven was. Recall Newton's law of cooling, $T'(t) = k(A - T(t))$. Suppose here $k = 2$. How long until the pie cools to half the temperature of the oven?[1]

[1] Your answer should have a natural logarithm in it. Calculators will not be allowed on the exam.