MTH 311 Sections C and F

# Final Exam Practice

Fall 2019

Here are some practice problems for the final exam. Be sure to also study the homeworks and previous exams.

1. Find the general solution to $\vec{x}' = A\vec{x}$ where the matrix $A$ is given by:

   (a) $\begin{bmatrix} 5 & 6 \\ -2 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 3/4 & -1 \end{bmatrix}$

   (d) $\begin{bmatrix} -1 & -5 \\ 1 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

2. Draw the phase portraits for the linear systems of 1 (a)–(e).

3. Consider the following non-linear system:

$$\begin{cases} x' = -4x + 4xy \\ y' = 2y - y^2 - xy \end{cases}$$

   (a) Find the equilibrium points.
   (b) Near each equilibrium sketch the phase portrait of the linearized sytem.
   (c) Use the information from (a) and (b) to sketch the total phase portrait.

4. Carry out (a)–(c) of Problem 3 for the following non-linear system:

$$\begin{cases} x' = 2x - 2x^2 + 5xy \\ y' = y - 2y^2 + 2xy \end{cases}$$

5. Use the Laplace Transform to solve the following initial value problems:

   (a) $y'' - 2y' - 3y = 0$, $y(0) = 2$, $y'(0) = 0$
   (b) $y'' + y = 2\sin(t)$, $y(0) = y'(0) = 0$
   (c) $y'' + 5y' + 4y = 1 - u_2(t)$, $y(0) = y'(0) = 0$
   (d) $y'' + 9y = \delta(t - 3\pi) + \cos(3t)$, $y(0) = y'(0) = 0$