Midterm 2 solutions

MTH 311, Sections C and F
Fall 2019

Name and ID number: ______________________________

No calculators, phones or any other devices may be present during the exam. Put them away. Show your work to receive full credit. The exam you took may have problems which are slightly different than the ones listed here.

1. (18 pts) Find the general solutions for the following differential equations.

   (a) $y'' - y' - 12y = 0$ $\quad c_1 e^{4t} + c_2 e^{-3t}$

   (b) $y'' - 2y' + 10y = 0$ $\quad c_1 e^t \cos(3t) + c_2 e^t \sin(3t)$

   (c) $D^2(D^2+3)^2 y = 0$ where $D = \frac{d}{dt}$
   $c_1 + c_2 t + c_3 \cos(\sqrt{3}t) + c_4 \sin(\sqrt{3}t) + tc_5 \cos(\sqrt{3}t) + c_6 t \sin(\sqrt{3}t)$

2. (18 pts) For the following equations, make a simplified guess for the form of the particular solution. Do not solve for the coefficients.

   (a) $y'' - y = \cos(t) - 2$ $\quad c_1 \cos(t) + c_2 \sin(t) + c_3$

   (b) $y^{(3)} + 4y' = \sin(2t) - 1$ $\quad c_1 t \cos(2t) + c_2 t \sin(2t) + c_3 t$

   (c) $y'' + 6y' + 9y = e^{-3t}$ $\quad c_1 t^2 e^{-3t}$

3. (16 pts) Use variation of parameters to find the general solution to

$$y'' - 2y' + y = \frac{e^t}{\sqrt{t}} \qquad (t > 0)$$

   (Useful: $W = W(y_1, y_2) = y_1 y_2' - y_2 y_1'$, $u_1 = -\int y_2 f/W$, $u_2 = \int y_1 f/W$.)
   $y_p(t) = c_1 e^t + c_2 t e^t + \frac{4}{3} t^{3/2} e^t$

4. (16 pts) Solve $y'' + 2y' + 3y = 0$ where $y(0) = 0$, $y'(0) = 1$.
   $y(t) = \frac{1}{\sqrt{2}} e^{-t} \sin(\sqrt{2}t)$

5. (16 pts) Consider a spring system with mass $m = 2$, spring constant $k = 12$, shock absorber with damping constant $b = 10$, and external force $f(t) = t\cos(t)$.

   (a) Set up the ODE for the motion of the mass $x(t)$.
   $2x'' + 10x' + 12x = t\cos(t)$

   (b) Write the general solution, without solving for the coefficients of $x_p(t)$.
   $x(t) = x_p(t) + c_1 e^{-2t} + c_2 e^{-3t}$, where
   $x_p(t) = \tilde{c}_1 \cos(t) + \tilde{c}_2 \sin(t) + \tilde{c}_3 t \cos(t) + \tilde{c}_4 t \sin(t)$

6. (16 pts) Solve $y'' + y = \cos(t)$ with initial conditions $y(0) = 1$, $y'(0) = -1$.
   $y(t) = \cos(t) - \sin(t) + \frac{1}{2} t \sin(t)$