**Toric mirrors**

Let $\Sigma$ be a fan with primitive vectors $v_1, \ldots, v_n$. Mirror to the toric variety $T_\Sigma$ is LG model $((\mathbb{C}^*)^n, W = \sum_j z^j)$. Following Bondal, [FLTZ] conjecture: Relative skeleton $L_{\Sigma|T}$ is equal to $L = \bigcup_{\sigma \in \Sigma} \sigma \times T^*T = (\mathbb{C}^*)^n - \mathbb{C}^n$.

Mirror symmetry for hypersurfaces

A generic hypersurface $H_\Delta \subset (\mathbb{C}^*)^n$ is determined by its Newton polytope $\Delta = \text{Conv}(v_1, \ldots, v_n) \subset \mathbb{R}^n$. This is a “large-volume limit.” Its mirror is the large-complex-structure limit of the toric stack $T_\Sigma$, where the fan $\Sigma_\Delta$ has primitive rays $\{v_1, \ldots, v_n\}$; in other words, $H_\Delta \subset (\mathbb{C}^*)^n$ is mirror to the toric boundary $\partial T_\Sigma$.

We prove this homological mirror symmetry equivalence.

**Theorem (Mirror symmetry for hypersurfaces)**

With notation as above, there is an equivalence of categories $\text{Fuk}(H_\Delta) \cong \text{Coh}(T_{\Sigma_\Delta})$.

Since we allow $T_\Sigma$ to be a (Deligne-Mumford) stack, the theorem applies to all hypersurfaces (including general type)!

**The proof of mirror symmetry**

Toric boundary $\partial T_\Sigma = (\text{toric varieties, glued along toric varieties})$.

The construction from [M] allows us to match this on the mirror: The skeleton $L_\Sigma = (\text{skeleta of mirrors to toric varieties, glued along skeleta of mirrors to toric varieties})$.

We can apply descent as soon as we know what the mirror to pushforward is.

**Lemma [G-Shende]:** Pushforward of orbisheaves is mirror to microlocalization.

Now we apply Kuwagaki’s theorem to match up pieces and observe that both sides are the same colimit.

$\text{Coh}(T_{\Sigma_\Delta}) = \text{colim}_{\Sigma_\Delta \subset \Sigma} \text{Coh}(T_{\Sigma_\Delta}) \cong \text{colim}_{\Sigma_\Delta \subset \Sigma} \text{Fuk}(W_{\Sigma_\Delta}) = \text{Fuk}(L_\Sigma)$.

**Moral**

If you can produce a nice skeleton, mirror symmetry becomes easy.

**Further directions**

- **Toric degenerations:** toric varieties glued together along toric varieties should have nice mirror skeleta.
- **New functorialities:** expected from mirror-symmetry structure: cf. [Au].
- **New calculations:** for symplectic resolutions.

**References**

[Au] Speculations on homological mirror symmetry for hypersurfaces in $(\mathbb{C}^*)^n$.


[GS] B. Gammage and V. Shende, Mirror symmetry for very affine hypersurfaces.


