SAMPLE MIDTERM 1 MATH 210 (S19)

Name:

Problem 1: (18 points) Write the general solution of the system below in parametric **vector** form.

$$\begin{bmatrix} 1 & 2 & 3 & 5 & 4 \\ 2 & 4 & 6 & 9 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Problem 2: (18 points) Let $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 1 & 3 \\ 3 & 6 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 & 1 & 3 \\ 3 & 6 & 1 & 4 \\ 4 & 8 & 2 & 6 \end{bmatrix}$.

a) Are A and B row equivalent? Justify with calculations.

b) Give an examples of 3 2×2 matrices A_1, A_2 and A_3 such that no pair has the same pivot positions, the columns of A_1 span $\mathbf{R}^2$, the system $A_2\mathbf{x} = \mathbf{b}$ has two free variables.

c) Consider a linear system $A\mathbf{x} = \mathbf{b}$ (A as above). Can such a system be inconsistent? Can such a system have a unique solution? Explain.

Problem 3: (18) a) Find a system of linear equations whose solution set is equal to Span($\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$).

b) Is $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ in Span($\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$)? Explain.

Problem 4: (18 points) Consider the vectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ and $\mathbf{v_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in $\mathbf{R}^3$.

a) Determine if the collection of vectors $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ is linearly independent. Show your calculations.

b) Give an example of a vector $\mathbf{v}$ in $\mathbf{R}^3$ such that the collection $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v}$ is linearly dependent. Explain the answer.

Problem 5: (18 points) Let $T_A : \mathbf{R^3} \to \mathbf{R^4}$ be the matrix transformation with $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 3 & 6 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

a) Find $T_A(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix})$ and $T_A(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})$.

b) Is T_A one-to-one? Explain.

c) Is T_A onto? Explain.

d) Find if possible a vector $\mathbf{v}$ such that $T\mathbf{v} = \mathbf{0}$. Explain.

Problem 6: (10 points) (**Attention** the corresponding question in the midterm will be very distinct from this one, but the same format) For the next questions determine if the statement is true or false. To get credit you need to explain your answers.

a) If the matrix equation $A\mathbf{x} = \mathbf{0}$ with A a 3x3 matrix has as solution set a plane, then A has 2 pivots.

b) The range of the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^4$ is spanned by two linearly independent vectors. Then there is a nonzero vector $\mathbf{v}$ in $\mathbf{R}^3$ such that $T\mathbf{v} = \mathbf{0}$.