MIDTERM 1 MATH 210 (FALL 2013)

Name:

Problem 1: (22 points) Write in parametric vector form the solution space to:

$$\begin{bmatrix} 1 & 2 & -1 & 3 & 5 \\ 2 & 4 & -2 & 6 & 9 \\ -1 & -2 & 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

Problem 2: (22 points) Balance the chemical equation:

$$x_1 Zn + x_2 NO_3 H = x_3 (NO_3)_2 Zn + x_4 H_2 O + x_5 N_2$$

Problem 3: (22 points) Consider the collection $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$ of vectors in $\mathbf{R}^4$.

a) Is the collection linearly independent? Explain.

b) Find which vectors (b_1, b_2, b_3, b_4) are in the span of the collection (that is find the equations the entries b_i must satisfy so that the vector (b_1, b_2, b_3, b_4) belongs to the span).

Problem 4: (22 points) Let $T : \mathbf{R^4} \to \mathbf{R^4}$ be the linear transformation given by $T(x, y, z, t) = (x + 2y + 3z + 4t, y + 3z + t, 3x + 2y + 3z + t, x - 3z + 2t)$.

a) Find the standard matrix A_T for T.

b) Are the columns of A_T linear independent? Explain.

c) Give an example of a $\mathbf{b} \in \mathbf{R^4}$ such that $T\mathbf{x} = \mathbf{b}$ has no solutions.

Problem 5: (12 points) For the next questions determine if the statement is true or false. To get credit you need to explain your answers.

a) Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a collection of vectors in $\mathbf{R}^3$ such that Span$\{\mathbf{v}_1, \mathbf{v}_2\}$=Span$\{\mathbf{v}_1, \mathbf{v}_3\}$. The collection $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linear dependent.

b) If the number of free variables of the linear system $A\mathbf{x} = \mathbf{b}$ is 2, where A is a 3×3 matrix, then the range of matrix transformation T_A associated to A is a plane in $\mathbf{R}^3$.

c) Let A be a 4×4 matrix whose columns are linearly independent. All linear systems $A\mathbf{x} = \mathbf{b}$ with $\mathbf{b} \in \mathbf{R}^4$ have one and only one solution.