Math 592 (The Math of Music) **Spring 2015**
Quiz 2 **Drew Armstrong**

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1. Write down "Euler's Formula".

$$e^{it} = \cos t + i \sin t \qquad \text{OR} \qquad \exp\begin{pmatrix} 0 & -t \\ t & 0 \end{pmatrix} = \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}.$$

2. Write down the trigonometric identity that explains the phenomenon of beats. [Hint: $\sin(u) + \sin(v) = \cdots$]

$$\sin(u) + \sin(v) = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

3. If two pure sine waves with frequencies 440 Hz and 444 Hz are played together, what do you expect to hear?

Note that

$$\sin(440 \cdot 2\pi t) + \sin(444 \cdot 2\pi t) = 2 \cdot \sin(442 \cdot 2\pi t) \cos(2 \cdot 2\pi t).$$

This will sound like a pure tone of frequency 442 Hz turning on and off 4 times per second (i.e., the beats have frequency 4 Hz).

4. List all of the reduced fractions between 1 and 2 in which the numerator and denominator are both less than or equal to 7.

$$1 < \frac{7}{6} < \frac{6}{5} < \frac{5}{4} < \frac{4}{3} < \frac{7}{5} < \frac{3}{3} < \frac{5}{3} < \frac{7}{4} < 2$$

5. This is the Plomp-Levelt dissonance curve based on seven partials. Label the cusps with the fractions you wrote down in Problem 4.

Voilà, the cusps are labeled: