Math 461 **Spring 2015**
Homework 6 **Drew Armstrong**

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1. Symmetric Functions. Consider the elementary symmetric functions

$$\begin{aligned} e_1 &= r + s + t \\ e_2 &= rs + rt + st \\ e_3 &= rst. \end{aligned}$$

They are called elementary because every other symmetric function can be expressed in terms of them. Express the following symmetric functions in terms of e_1, e_2, e_3.

(a) $r^2 + s^2 + t^2$
(b) $r^3 + s^2 + t^3$
(c) $\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2}$

2. Application. Suppose that the polynomial x^3+px+q has roots r, s, t. Find the polynomial (with leading coefficient 1) whose roots are rs, rt, st. The coefficients must be expressed in terms of p and q. [Hint: The polynomial is $(x - rs)(x - rt)(x - st)$.]

3. Discrete Fourier Transform. Let $\omega = 2^{2\pi i/3}$. In class I claimed that

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}.$$

Verify that this is true.

4. Example of Lagrange's Method. In this problem we will find the full solution to the cubic equation $x^3 - 6x - 6 = 0$. [Compare to Exam 1 Problem 3.] Let r_1, r_2, r_3 be the three complex solutions and note that $e_1 = r_1 + r_2 + r_3 = 0$, $e_2 = r_1r_2 + r_1r_3 + r_2r_3 = -6$, $e_3 = r_1r_2r_3 = 6$. Let $\omega = e^{2\pi i/3}$, and define

$$\begin{aligned} s_1 &= r_1 + r_2 + r_3 \\ s_2 &= r_1 + \omega r_2 + \omega^2 r_3 \\ s_3 &= r_1 + \omega^2 r_2 + \omega r_3. \end{aligned}$$

(a) We saw in class that $s_2^3 + s_3^3 = 2e_1^3 - 9e_1e_2 + 27e_3 = 162$ and $s_2s_3 = e_1^2 - 3e_2 = 18$. Use this information to compute the values of s_2^3 and s_3^3.
(b) Let s_2 and s_3 be the positive real cube roots of the values for s_2^3 and s_3^3 that you computed above (it doesn't matter which values we choose, so we might as well choose the easiest ones). Now use the result from Problem 3 to find explicit formulas for the three roots r_1, r_2, r_3.