Math 461 **Spring 2015**
Homework 3 **Drew Armstrong**

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1. De Moivre's Theorem.

(a) Use de Moivre's Theorem to express $\cos(2\theta)$ as a polynomial in $\cos(\theta)$.
(b) Solve this polynomial to obtain a formula for $\cos(\theta)$ in terms of $\cos(2\theta)$.
(c) Use the formula from (b) to find the exact value of $\cos(\pi/8)$.

2. Quadratic Formula Again.

(a) Compute the square roots of i.
(b) Use part (a) to solve the equation $\frac{1}{2}z^2 + (1+i)z + \frac{i}{2} = 0$ for $z \in \mathbb{C}$.

3. Complex Conjugation. Recall that complex conjgation $* : \mathbb{C} \to \mathbb{C}$ is defined by

$$(a+ib)^* := a - ib.$$

Show that for all $u, v \in \mathbb{C}$ we have

(a) $(u+v)^* = u^* + v^*$
(b) $(uv)^* = u^*v^*$
(c) $|u||v| = |uv|$. [Hint: $|u|^2 = uu^*$.]

4. Conjugate Pairs of Roots.

(a) Consider a polynomial with **real** coefficients, $f(x) \in \mathbb{R}[x]$. Show that for all **complex** numbers $z \in \mathbb{C}$ we have $f(z)^* = f(z^*)$.
(b) Conclude that the **complex** roots of a **real** polynomial come in conjugate pairs.

5. Useful Little Theorem. Let $f(x)$ be a polynomial of degree 3 with **real** coefficients. Prove that if $f(x)$ has a **complex** root, then it must also have a **real** root. [Hint: If $f(u) = 0$ for some $u \in \mathbb{C}$, show that $f(x)$ is divisible by $(x^2 - (u+u^*)x + uu^*)$. Show that the quotient must have real coefficients.]