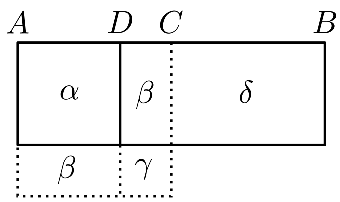
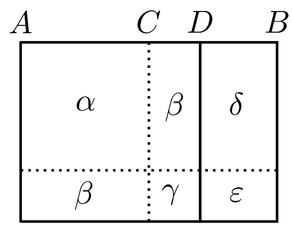
Math 461 **Spring 2015**
Homework 1 **Drew Armstrong**

1. In al-Khwarizmi's solution of quadratic equations he needed to solve the following geometric problem. Consider a line segment AB. Let C be its midpoint and let D be any other point on the segment. Construct a square on AD and complete this to a rectangle on AB. There are two different ways this could look (see the solid lines):





In both cases give a geometric argument that the area of the solid rectangle on DB plus the area of the square on CD equals the area of the square on AC. [Hint: Divide the diagrams by the suggested dotted lines. The Greek letters represent different areas in the two diagrams.]

2. Consider the quadratic equation $(x - r)(x - s) = 0$, where r and s are constants.

(a) Show that the discriminant of this equation is $(r - s)^2$.
(b) Show that the discriminant is zero if and only if $r = s$.

3. Suppose that the quadratic equation $x^2 + px + q = 0$ has solutions $x = r$ and $x = s$. Find a quadratic equation with solutions $x = 1/r$ and $x = 1/s$. [Hint: Use $(x-r)(x-s) = x^2+px+q$ to express p and q in terms of r and s. Now consider $(x - 1/r)(x - 1/s)$.]

4. Factor the following cubic polynomials as $f(x) = (x - r)(x - s)(x - t)$ by: (1) guessing a solution to $f(x) = 0$, (2) using long division, (3) using the quadratic formula.

(a) $f(x) = x^3 - 3x^2 + x + 1$
(b) $f(x) = x^3 - 1$

5. Consider the following diagram from Descartes' *La Géométrie* (1637). Prove that the distances MQ and MR are solutions to the quadratic equation $y^2 + b^2 = ay$.