**Math 461 F** **Spring 2011**
**Homework 2** **Drew Armstrong**

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**Reading.**

Sections 2.1 and 2.2

**Problems.**

**A.1.** Suppose that the cubic equation $ax^3 + bx^2 + cx + d = 0$ has three roots, called $r, s, t$. Give a formula for $rs + rt + st$ in terms of $a, b, c, d$.

**A.2.** Find all complex solutions $z \in \mathbb{C}$ to the quadratic equation

$$z^2 - z + \left(\frac{1}{4} - \frac{i}{2}\right) = 0.$$

**A.3.** Use de Moivre's formula and the fact that $\cos^2\alpha + \sin^2\alpha = 1$ for all $\alpha \in \mathbb{R}$ to come up with a formula for $\cos(\theta/2)$ in terms of $\cos\theta$ alone. (You can assume $\cos(\theta/2) \geq 0$.) Use your formula to find the **exact** value of $\cos(\pi/8)$.

**A.4.** Let $\omega = \cos(2\pi/3) + i\sin(2\pi/3)$. Prove that for any $a, b$ we have

$$a^3 - b^3 = (a - b)(a - \omega b)(a - \omega^2 b).$$

Can you find a similar formula for the difference $a^n - b^n$ of $n$th powers? Hint: Factor $x^n - 1$ and then put $x = a/b$.

**A.5.** Prove that for every positive integer $n > 1$ we have

$$\sum_{k=1}^{n} \cos\frac{2\pi k}{n} = 0.$$

Hint: Consider the number $\omega = \cos(2\pi/n) + i\sin(2\pi/n)$.

**A.6.** Define a function $f : \mathbb{C} \to \mathrm{M}_{2\times 2}(\mathbb{R})$ from the complex numbers to the $2 \times 2$ real matrices by setting

$$f(a + ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}.$$

For any complex numbers $z, w \in \mathbb{C}$ verify the following:

(a) $f(z + w) = f(z) + f(w)$,
(b) $f(zw) = f(z)f(w)$,
(c) $|z|^2 = \det f(z)$.

(The operations on the right hand sides of the equations are matrix addition, matrix multiplication, and matrix determinant.)