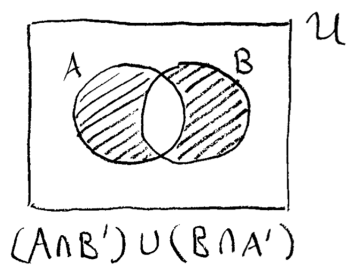
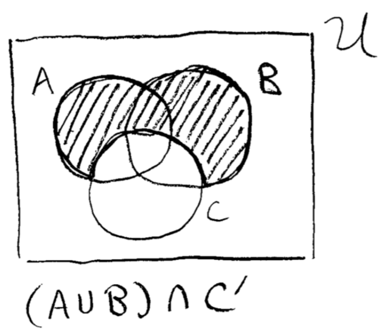
Math 309 **Summer 2019**
Quiz 2 **Drew Armstrong**

1. Venn Diagrams. Let $A, B, C \subseteq U$ be subsets of the universal set.

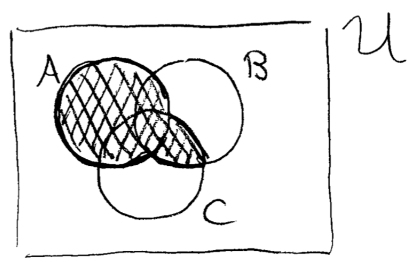
(a) Draw a Venn diagram to show the set $(A \cap B') \cup (B \cap A')$.



(b) Draw a Venn diagram to show the set $(A \cup B) \cap C'$.



(c) Tell the name of the following set (many correct answers):



The shortest name of the set is $A \cup (B \cap C)$. The disjunctive normal form is

$$(A \cap B \cap C) \cup (A \cap B' \cap C) \cup (A \cap B \cap C') \cup (A \cap B' \cap C') \cup (A' \cap B \cap C).$$

Or we can compute the disjunctive normal form of the unshaded region. Then we take the complement and apply de Morgan's law:

$$\begin{aligned}&\big[(A' \cap B' \cap C) \cup (A' \cap B \cap C') \cup (A' \cap B' \cap C')\big]'\\&= (A' \cap B' \cap C)' \cap (A' \cap B \cap C')' \cap (A' \cap B' \cap C')'\\&= (A \cup B \cup C') \cap (A \cup B' \cup C) \cap (A \cup B \cup C).\end{aligned}$$

The resulting expression is called the *conjunctive normal form.*

2. Boolean Functions. Consider the Cartesian product set:

$$\{T,F\}^n := \underbrace{\{T,F\} \times \{T,F\} \times \cdots \times \{T,F\}}_{n \text{ times}}.$$

(a) Tell me the number of elements of the set $\{T,F\}^n$.

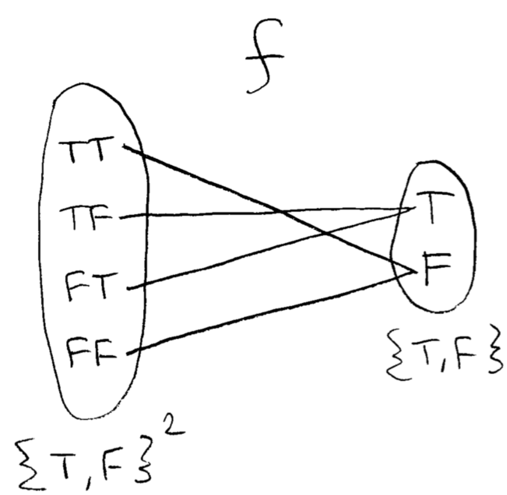
$$\#\{T,F\}^n = \underbrace{\#\{T,F\} \times \#\{T,F\} \times \cdots \times \#\{T,F\}}_{n \text{ times}} = \underbrace{2 \times 2 \times \cdots \times 2}_{n \text{ times}} = 2^n$$

(b) Count the functions $f : \{T,F\}^2 \to \{T,F\}$ with two inputs and one output.

The number of functions from A and B is $(\#B)^{\#A}$. Hence the number of functions from $\{T,F\}^2$ to $\{T,F\}$ is

$$(\#\{T,F\})^{(\#\{T,F\}^2)} = 2^{(2^2)} = 2^4 = 16.$$

(c) Fill in the arrows for the function $f(P,Q) = (P \wedge \neg Q) \vee (Q \wedge \neg P)$:



Remark: This is just one of the 16 Boolean functions with two inputs and one output. It is commonly called XOR. This function also appeared in Problem 1(a).