Math 309 **Summer 2019**
Quiz 1 **Drew Armstrong**

Problem 1.

(a) Draw Pascal's Triangle down to the 7th row.

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}
$$

(b) Use the triangle to expand $(1+x)^7$.

$$(1+x)^7 = 1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7$$

(c) Use the triangle to evaluate the following sum:

$$
\begin{aligned}
\sum_{k=0}^{4} (-1)^k \binom{7}{k} &= \binom{7}{0} - \binom{7}{1} + \binom{7}{2} - \binom{7}{3} + \binom{7}{4} \\
&= 1 - 7 + 21 - 35 + 35 \\
&= 15
\end{aligned}
$$

Problem 2. Let the sequence $S_0, S_1, S_2, \ldots$ be defined by the following initial condition and recurrence relation:

$$S_n := \begin{cases} 1 & \text{if } n = 0, \\ S_{n-1} + 2^{n-1} & \text{if } n \geq 1. \end{cases}$$

(a) Fill in the following table:

n	0	1	2	3	4	5
S_n	1	2	4	8	16	32

(b) Try to guess a simple formula for S_n.

I guess that $S_n = 2^n$ for all $n \geq 0$.

(c) Use induction to prove that your formula is correct.

- *Base Case.* If $n = 0$ then we have $S_0 = 1 = 2^0$. ✓
- *Induction Step.* Now fix some $n \geq 0$ and assume for induction that $S_n = 2^n$. In this case we want to prove that $S_{n+1} = 2^{n+1}$. Indeed, we observe that

$$\begin{aligned} S_{n+1} &= S_n + 2^n && \text{by definition} \\ &= 2^n + 2^n && \text{by assumption} \\ &= 2 \cdot 2^n \\ &= 2^{n+1}. \end{aligned}$$