**Math 309** **Summer B 2018**
**Quiz 2** **Drew Armstrong**

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**0. (Make-up for Quiz 1)** The numbers $p_n$ are defined by the initial condition $p_0 = 1$ and the recurrence $p_n = p_{n-1} - n$ for all $n \geq 1$. Find a closed formula:

$$\begin{aligned} p_n &= 1 - 1 - 2 - 3 - \cdots - n \\ &= 1 - (1 + 2 + 3 + \cdots + n) \\ &= 1 - \frac{n(n+1)}{2} \end{aligned}$$

**1.** Let $S$ be a set and for all elements $x \in S$ let $P(x)$ be a logical statement. Translate the following statements into English:

- "$\forall x \in S, P(x)$"

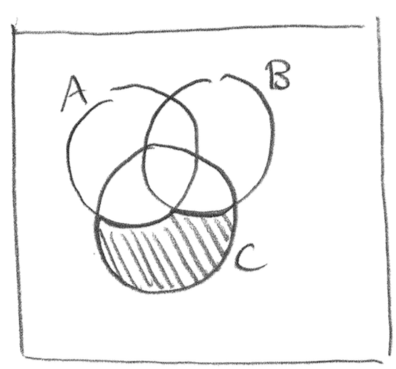
"For all elements $x$ in $S$, the statement $P(x)$ is true."
or "The statement $P(x)$ holds for every element $x$ in $S$."
or "Every element $x$ of $S$ satisfies $P(x)$."

- "$\exists x \in S, \neg P(x)$"

"There exists an element $x$ in $S$ such that $P(x)$ is false."

**2.** Let $A, B, C$ be subsets of the universal set $U$. Use a Venn diagram to illustrate the set

$$(A \cup B)^c \cap C.$$



**3.** Complete the following truth table:

| $P$ | $Q$ | $P \wedge Q$ | $P \vee Q$ | $P \Rightarrow Q$ |
|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $F$ |
| $F$ | $T$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $F$ | $T$ |

**4.** Consider the Boolean function $\varphi(P, Q)$ defined as follows:

| $P$ | $Q$ | $\varphi(P, Q)$ |
|---|---|---|
| $T$ | $T$ | $F$ |
| $T$ | $F$ | $T$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $T$ |

Write down two different algebraic formulas for this function:

(There are infinitely many correct answers.)

$$\varphi(P, Q) = (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$$

$$\varphi(P, Q) = \neg Q$$