Math 309 **Summer B 2018**
Quiz 1 **Drew Armstrong**

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1. Let n be a positive integer. Tell me a **closed formula** for the following sum:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

2. Use your answer from Problem 1 to simplify the following sum:

$$\sum_{k=1}^{n}(2k+1) = 2\left(\sum_{k=1}^{n} k\right) + \left(\sum_{k=1}^{n} 1\right) = 2 \cdot \frac{n(n+1)}{2} + n = n(n+1) + n = n(n+2)$$

3. Suppose that the numbers p_n are defined by the initial condition $p_0 = 1$ and the recurrence $p_{n+1} = p_n + n + 2$ for all $n \geq 0$. Fill in the following table:

n	0	1	2	3	4
p_n	1	3	6	10	15

4. Continuing from Problem 3, use your answer from Problem 1 to find a **closed formula**:
There are many ways to do this. Here's the least clever way:

$$\begin{aligned}
p_0 &= 1 \\
p_1 &= 1 + (0+2) \\
p_2 &= 1 + (0+2) + (1+2) \\
&\vdots \\
p_n &= 1 + (0+2) + (1+2) + (3+2) + (4+2) + \cdots + ((n-1)+2) \\
&= 1 + \sum_{k=0}^{n-1}(k+2) \\
&= 1 + \left(\sum_{k=0}^{n-1} k\right) + \left(\sum_{k=0}^{n-1} 2\right) \\
&= 1 + \frac{(n-1)n}{2} + 2n \\
&= \frac{2 + (n-1)n + 4n}{2} \\
&= \frac{n^2 + 3n + 2}{2} \\
&= \frac{(n+1)(n+2)}{2}
\end{aligned}$$