Math 309 — Summer 2014
Quiz 3 Solutions — Drew Armstrong

1. Accurately state the Binomial Theorem.

Proof. For any complex numbers $x, y \in \mathbb{C}$ and any natural number $n \in \mathbb{N}$ we have

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

□

2. Explicitly write out the expansion of $(1+x)^6$. [Evaluate all binomial coefficients.]

Proof. The quickest way to compute the coefficients is by drawing Pascal's triangle. We get

$$(1+x)^6 = 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6.$$

□

3. Let S be a set with 6 elements. How many subsets of size 3 does S have?

Proof. The answer is

$$\binom{6}{3} = \frac{6!}{3!\,3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20.$$

□

4. You flipped a **fair coin** 6 times. What is the probability that you got "heads" exactly 3 times?

Proof. The answer is

$$\frac{\binom{6}{3}}{2^6} = \frac{20}{64} = 31.25\%.$$

□

5. You flipped a **biased coin** 6 times (assume $P(\text{"heads"}) = 2/3$ and $P(\text{"tails"}) = 1/3$). What is the probability that you got "heads" exactly 3 times?

Proof. The answer is

$$\binom{6}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 = 20 \cdot \frac{2^3 \cdot 1^3}{3^6} = \frac{160}{729} \approx 21.95\%.$$

□