Math 309 Summer 2014
Homework 4 Drew Armstrong

Recall that we define the logical symbol "$\Rightarrow$" by "$P \Rightarrow Q$" := "$\neg P \vee Q$". We pronounce the statement "$P \Rightarrow Q$" as "if P then Q", or "P implies Q". Recall that we say an integer $n \in \mathbb{Z}$ is **even** if there exists $k \in \mathbb{Z}$ such tha $n = 2k$, and we say that $n \in \mathbb{Z}$ is **odd** if there exists $k \in \mathbb{Z}$ such that $n = 2k+1$. On Problems 1–4 you will give a (hopefully) nice proof that for all $m, n \in \mathbb{Z}$ we have

$$(mn \text{ is even}) \Longleftrightarrow (m \text{ is even or } n \text{ is even}).$$

1. Use the contrapositive to show that "$P \Rightarrow (Q \vee R)$" = "$(\neg Q \wedge \neg R) \Rightarrow \neg P$".

2. Use the result from Problem 1 to prove that if mn is even, then m is even or n is even. [Hint: Let $P =$"mn is even", $Q =$"m is even", and $R =$"n is even".]

3. Use a truth table to show that "$(P \vee Q) \Rightarrow R$" = "$(P \Rightarrow R) \wedge (Q \Rightarrow R)$".

4. Use the result from Problem 3 to prove that if m is even or n is even, then mn is even. [Hint: What are P, Q, and R in this case?]

5. Fill in the rest of this table. What does it remind you of? Use the table to express the statement "$m+n$ is even" in terms of the statements "m is even" and "n is even", together with the logical symbols $\vee, \wedge, \neg$.

m	n	mn	$m+n$
E	E		
E	O		
O	E		
O	O		