Math 309 **Fall 2014**
Homework 3 **Drew Armstrong**

On this homework you will meet some new Boolean functions.

1. Given $P, Q \in \{T, F\}$ we define the **Boolean sum** (also called "exclusive OR"):

$$P \oplus Q := (P \wedge \neg Q) \vee (\neg P \wedge Q).$$

(a) Draw the truth table for $P \oplus Q$.
(b) Use truth tables to prove that for all $P, Q, R \in \{T, F\}$ we have

$$P \wedge (Q \oplus R) = (P \wedge Q) \oplus (P \wedge R).$$

[It is fair to think of $\oplus$ as "addition" and $\wedge$ as "multiplication".]

2. Given $P, Q \in \{T, F\}$ we define the function $P \Rightarrow Q$ with the following table:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

We call this **logical implication** and we read $P \Rightarrow Q$ as "if P then Q" or "P implies Q".
(a) Draw the truth table for $P \not\Rightarrow Q := \neg(P \Rightarrow Q)$.
(b) Compute the disjunctive normal form of $P \not\Rightarrow Q$.
(c) Use part (b) to find a simple formula for $P \Rightarrow Q$. [Hint: De Morgan's Law.]

3. For all $P, Q \in \{T, F\}$ we define the function $P \Leftrightarrow Q$ by

$$P \Leftrightarrow Q := (P \Rightarrow Q) \wedge (Q \Rightarrow P).$$

We call this function **logical equivalence** and we read $P \Leftrightarrow Q$ as "P if and only if Q".
(a) Compute the disjunctive normal form of $P \Leftrightarrow Q$.
(b) Show that $P \not\Leftrightarrow Q := \neg(P \Leftrightarrow Q)$ is the same as $P \oplus Q$.

4. Let B be a Boolean algebra. For all $P, Q \in B$ we define the "Sheffer stroke"

$$P \uparrow Q := \neg(P \wedge Q).$$

Use the properties of Boolean algebra from the handout to prove the following formulas. Don't use truth tables! These formulas can be used to express **any** function $\{T, F\}^n \to \{T, F\}$ in terms of $\uparrow$ alone.
(a) $\neg P = P \uparrow P$
(b) $P \vee Q = (P \uparrow P) \uparrow (Q \uparrow Q)$
(c) $P \wedge Q = (P \uparrow Q) \uparrow (P \uparrow Q)$