Math 309 **Fall 2014**
Homework 2 **Drew Armstrong**

If S is a **finite** set, we let $\#S$ denote its number of elements. We call this the **size** or the **cardinality** of S. Sometimes we use the equivalent notation $|S| := \#S$.

1. Let X and Y be **finite** sets and let $f : X \to Y$ be a function.

(a) We say that $f : X \to Y$ is an **injection** if for all $y \in Y$ there is at *most* one $x \in X$ such that $f(x) = y$. If $f : X \to Y$ is an injection, show that $\#X \leq \#Y$.

(b) We say that $f : X \to Y$ is a **surjection** if for all $y \in Y$ there is at *least* one $x \in X$ such that $f(x) = y$. If $f : X \to Y$ is a surjection, show that $\#X \geq \#Y$.

(c) We say that $f : X \to Y$ is a **bijection** if it is both an injection and a surjection. If $f : X \to Y$ is a bijection, show that $\#X = \#Y$.

[Hint: For each $y \in Y$ let $d(y)$ denote the number of $x \in X$ such that $f(x) = y$. What can you say about the sum $\sum_{y \in Y} d(y)$?]

2. If X and Y are finite sets, explain why there are $\#Y^{\#X}$ different functions from X to Y.

3. Explicitly write down all of the functions from $\{1, 2, 3\}$ to $\{T, F\}$. How many are there? (See Problem 2.) How many of these functions are injective, surjective, bijective?

4. Explicitly write down all of the subsets of $\{1, 2, 3\}$. Compare to your answer to Problem 3. Can you describe a bijection (one-to-one correspondence) between the set of functions $\{1, 2, 3\} \to \{T, F\}$ and the set of subsets of $\{1, 2, 3\}$?

5. How many functions are there from $\{1, 2, 3\}$ to $\{1, 2, 3\}$? (Don't write them all down.) How many of the functions $\{1, 2, 3\} \to \{1, 2, 3\}$ are **bijections**? Explicitly write them down.