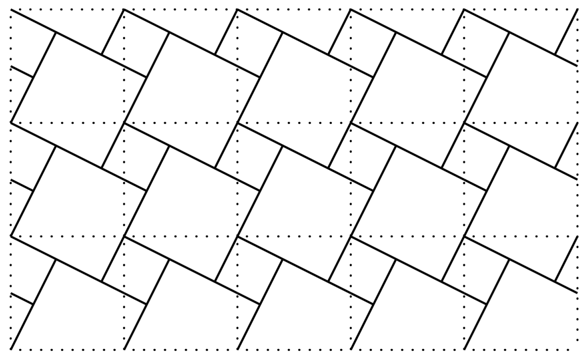
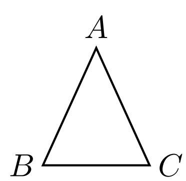
Math 230 | **Spring 2019**
Homework 1 | **Drew Armstrong**

Problem 1. Here is a geometric picture proof of the Pythagorean theorem:



Your job is to **explain the proof** at an informal level. For example, imagine that you are tutoring a high school student. It might be helpful to label the triangle and the three side lengths, but please **don't use algebra**. [Hint: What is the area of the dotted square?]

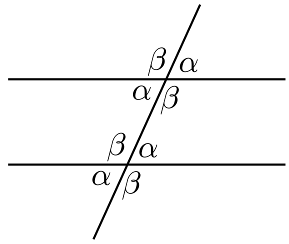
Problem 2. Proposition I.5 in Euclid has aquired the name *pons asinorum* (the "bridge of asses" or "bridge of fools"). Apparently, many students never got past this point in their studies. The proposition says the following: Consider a triangle $\triangle ABC$. If the side lengths $\overline{AB}$ and $\overline{AC}$ are equal, then the angles $\measuredangle ABC$ and $\measuredangle ACB$ are equal:



$$\boxed{\overline{AB} = \overline{AC} \quad \Longrightarrow \quad \measuredangle ABC = \measuredangle ACB}$$

(a) Look up Euclid's proof of Prop I.5 and try to understand it.
(b) **Write down the proof in your own words.** Your goal is to make the proof as understandable as possible. Maybe you can improve on Euclid.

Problem 3. Prove that the interior angles of any (Euclidean) triangle sum to 180°. You may use the following two facts without proof. **Prop I.31:** Given a line ℓ and a point p not on ℓ, **it is possible** to draw a line through p parallel to ℓ. **Prop I.29:** If a line falls on two parallel lines, then the corresponding angles are equal, as in the following figure:



[Hint: Your proof should begin as follows: "Consider a triangle with interior angles α, β, γ. We will prove that $\alpha + \beta + \gamma = 180°$." Now draw the triangle.]

Problem 4. The *dot product* of the vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ is defined by $\mathbf{u} \bullet \mathbf{v} := u_1 v_1 + u_2 v_2$. The *length* $\|\mathbf{u}\|$ of a vector $\mathbf{u}$ satisfies $\|\mathbf{u}\|^2 = \mathbf{u} \bullet \mathbf{u}$.

(a) The vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$ form the three sides of a triangle. Draw this triangle.

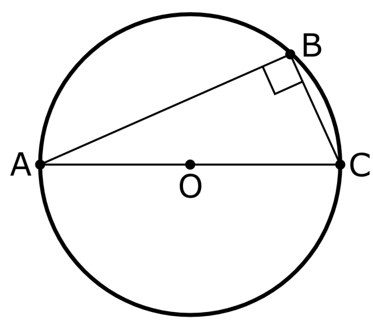
(b) Use algebra (not geometry) to prove that $\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(\mathbf{u} \bullet \mathbf{v})$.

(c) Use the formula from part (b) to prove the following statement:

"the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular if and only if $\mathbf{u} \bullet \mathbf{v} = 0$."

[Hint: Remember your picture from part (a). You are allowed to assume that the Pythagorean Theorem and its converse are true.]

Problem 5. Let AC be the diameter of a circle and let B be any other point of the circle. Then I claim that $\measuredangle ABC$ is a right angle:



Legend says that this is the oldest theorem in the world, and that it was proved by Thales of Alexandria in the 6th century BC. You will give a modern ("analytic") proof.

You can assume that $O = (0, 0)$ is at the origin of the Cartesian plane. You can also assume that $A = (-1, 0)$ and $C = (1, 0)$, so the circle has radius 1. Then the point B has the form $B = (\cos\theta, \sin\theta)$ for some angle θ. Now use Problem 4 to **prove that the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are perpendicular**. [Hint: Head minus tail. You may use any trig identities that you know.]