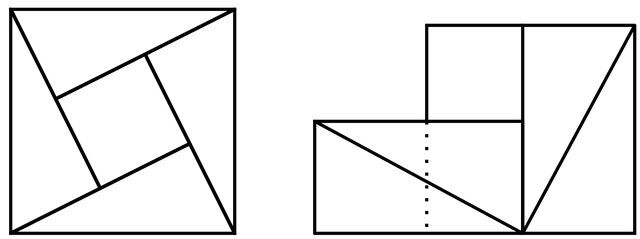
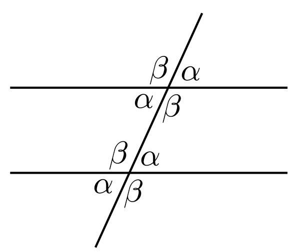
Math 230 — Fall 2018
Homework 1 — Drew Armstrong

Problem 1. In the *Lilavati*, the Indian mathematician Bhaskara (1114–1185) gave a one-word proof of the Pythagorean theorem. He said: **"Behold!"**



Add words to the proof. Your goal is to persuade a high school student who claims that he/she doesn't "get it." Try to avoid algebra as much as possible.

Problem 2. Prove that the interior angles of any triangle sum to 180°. You may use the following two facts without proof. **Prop I.31:** Given a line ℓ and a point p not on ℓ, **it is possible** to draw a line through p parallel to ℓ. **Prop I.29:** If a line falls on two parallel lines, then the corresponding angles are equal, as in the following figure:



Problem 3. Look up Euclid's Proposition I.48. Tell me the statement and the proof. Your goal is to make everything as understandable as possible, especially to yourself but also to me. Imagine that you are trying to teach this theorem to one of your classmates.

Problem 4. The dot product of the vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$ is defined by $\mathbf{u} \bullet \mathbf{v} := u_1 v_1 + u_2 v_2$. The length $\|\mathbf{u}\|$ of a vector $\mathbf{u}$ satisfies $\|\mathbf{u}\|^2 = \mathbf{u} \bullet \mathbf{u}$.

(a) The vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} - \mathbf{v}$ form the three sides of a triangle. Draw this triangle.

(b) Use algebra (not geometry) to prove that $\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(\mathbf{u} \bullet \mathbf{v})$.

(c) Use the formula from part (b) to prove the following statement:

"the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular if and only if $\mathbf{u} \bullet \mathbf{v} = 0$."

[Hint: Remember your picture from part (a). What do the Pythagorean Theorem and its converse tell you about this picture?]