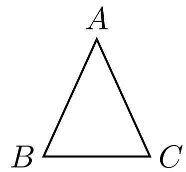
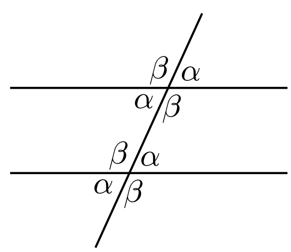
Math 230 E **Homework 1** **Fall 2013** **Drew Armstrong**

Problem 1. Proposition I.5 in Euclid has acquired the name *pons asinorum*, which translates as "bridge of asses" or "bridge of fools". Apparently, many students never got past this proposition. (I would say that the *pons asinorum* in today's curriculum is addition of fractions.) The proposition says the following: Consider a triangle $\triangle ABC$. **If** the side lengths $\overline{AB}$ and $\overline{AC}$ are equal (i.e. the triangle is *isoceles*), **then** the angles $\angle ABC$ and $\angle ACB$ are equal.

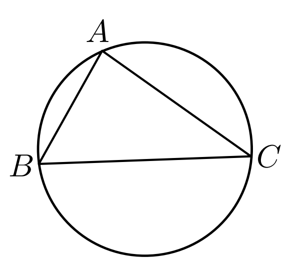


Your assignment is to **look up Euclid's proof and tell it to me.**

Problem 2. **Prove** that the interior angles of any triangle sum to 180°. You may use the following two facts without justification. **Fact 1:** Given a line ℓ and a point p not on ℓ, **it is possible** to draw a line through p parallel to ℓ. **Fact 2:** If a line falls on two parallel lines, then the corresponding angles are equal, as in the following figure.



Problem 3. **Prove** Thales' Theorem, which says the following. Consider a triangle $\triangle ABC$ inscribed in a circle. **If** line segment BC is a diameter of the circle, **then** angle $\angle BAC$ is a right angle. You may quote the results from Problems 1 and 2.



Problem 4. The dot product of vectors $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ and $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ is defined by $\mathbf{u} \cdot \mathbf{v} := u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$. The length $\|\mathbf{u}\|$ of a vector $\mathbf{u}$ is defined by $\|\mathbf{u}\|^2 := \mathbf{u} \cdot \mathbf{u}$.

(a) Prove the formula $\|\mathbf{u} - \mathbf{v}\|^{\mathbf{2}} = \|\mathbf{u}\|^{\mathbf{2}} + \|\mathbf{v}\|^{\mathbf{2}} - \mathbf{2}(\mathbf{u} \cdot \mathbf{v})$.

(b) Use this formula together with the 2D Pythagorean Theorem **and its converse** to prove the following statement:

"the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular if and only if $\mathbf{u} \cdot \mathbf{v} = 0$."

[Hint: Where is the triangle? Recall that you must prove both directions of the *if and only if* statement separately.]

Problem 5. Use vectors to give an analytic proof of Thales' Theorem. [Hint: You may assume that your circle is the unit circle in the Cartesian plane. You may assume that $B = (-1, 0)$, $C = (1, 0)$, and $A = (\cos\theta, \sin\theta)$ for some angle θ. Consider the vectors $\mathbf{u} = A - B$ and $\mathbf{v} = A - C$. Compute the dot product $\mathbf{u} \cdot \mathbf{v}$.]