Math 230 D **Fall 2012**
Homework 2 **Drew Armstrong**

Problem 1.

(a) Prove that a product of odd numbers is odd.
(b) Prove that 3^n is odd for all integers $n \geq 1$. [It's easy to prove that, say, 3^{101} is odd. But how will you prove it for infinitely many different n without having to say infinitely many things? I'll warn you: There is a big issue here called **induction**.]
(c) Assume for the moment that there exists a number x such that $2^x = 3$ and call it $x = \log_2(3)$. Prove that $\log_2(3)$ is not a fraction.

Problem 2. Prove that there is no perfect square of the form $4k+3$. That is, prove that there do not exist integers n and k such that $n^2 = 4k+3$.

Problem 3. Let P, Q and R be logical statements.

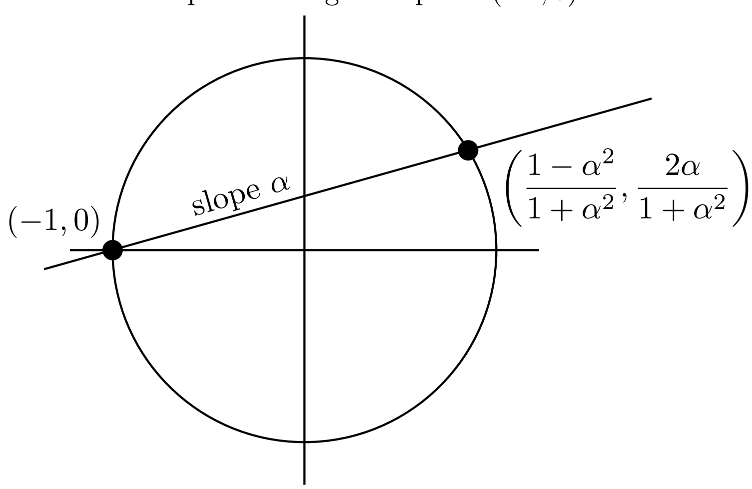
(a) Use a truth table to prove that the statement $P \Rightarrow (Q \text{ OR } R)$ is logically equivalent to the statement $(P \text{ AND NOT } Q) \Rightarrow R$.
(b) Use a truth table to prove that the statement $(P \text{ OR } Q) \Rightarrow R$ is logically equivalent to the statement $(P \Rightarrow R) \text{ AND } (Q \Rightarrow R)$.

Problem 4. Let m and n be integers. Prove that

$$\text{“}(m \text{ is even OR } n \text{ is even}) \Leftrightarrow mn \text{ is even”}.$$

Explicitly state any logical principles that you use. [Hint: You will need Problem 3.]

Problem 5. Draw a line of slope α through the point $(-1,0)$ on the unit circle.



(a) Prove that the other point of intersection is $\left(\frac{1-\alpha^2}{1+\alpha^2}, \frac{2\alpha}{1+\alpha^2}\right)$.
(b) Prove that α is a fraction if and only if $\frac{1-\alpha^2}{1+\alpha^2}$ and $\frac{2\alpha}{1+\alpha^2}$ are **both** fractions.

[Note: This gives us a **bijection** between the “rational points” on the circle (except for $(-1,0)$) and the set of all “rational numbers”.]