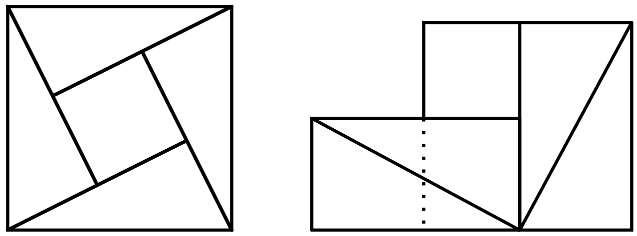
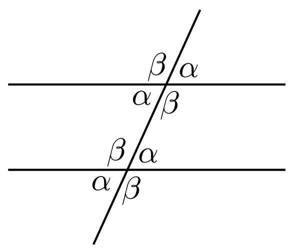
Math 230 D **Fall 2012**
Homework 1 **Drew Armstrong**

Problem 1. In the *Lilavati*, the Indian mathematician Bhaskara (1114–1185) gave a one-word proof of the Pythagorean theorem. He said: **"Behold!"**



Add words to the proof. Your goal is to persuade a high school student who claims he/she doesn't "get it". **Avoid algebra if possible.** (Sorry, the two pictures are not quite to scale.) [Hint: The dotted line is not in Bhaskara's figure. I added it as a suggestion.]

Problem 2. Prove that the interior angles of any triangle sum to 180°. You may use the following two facts without justification. **Fact 1:** Given a line ℓ and a point p not on ℓ, **it is possible** to draw a line through p parallel to ℓ. **Fact 2:** If a line falls on two parallel lines, then the corresponding angles are equal, as in the following figure.



Problem 3. Prove that $\sqrt{3}$ is not a fraction, in two steps.

(a) First **prove** the following lemma: Given a whole number n, if n^2 is a multiple of 3, then so is n. [Hint: Use the contrapositive, and note that there are two ways for n to be "**not** divisible" by 3.]

(b) Use the method of contradiction to **prove** that $\sqrt{3}$ is not a fraction. Quote your lemma in the proof.

Problem 4. Use the 2D Pythagorean Theorem to **prove** the 3D Pythagorean Theorem. That is, prove that the distance between points $(0,0,0)$ and (x,y,z) equals $\sqrt{x^2+y^2+z^2}$. [Hint: There are two triangles involved.]

Problem 5. The dot product of vectors $\mathbf{u} = (u_1, u_2, \ldots, u_n)$ and $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ is defined by $\mathbf{u}\cdot\mathbf{v} := u_1v_1 + u_2v_2 + \cdots + u_nv_n$. The length $\|\mathbf{u}\|$ of a vector $\mathbf{u}$ is defined by $\|\mathbf{u}\|^2 := \mathbf{u}\cdot\mathbf{u}$.

(a) Prove the formula $\|\mathbf{u}-\mathbf{v}\|^{\mathbf{2}} = \|\mathbf{u}\|^{\mathbf{2}} + \|\mathbf{v}\|^{\mathbf{2}} - \mathbf{2}(\mathbf{u}\cdot\mathbf{v})$.

(b) Use this formula together with the 2D Pythagorean Theorem **and its converse** to prove the following statement:

"the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular if and only if $\mathbf{u}\cdot\mathbf{v} = 0$."

[Hint: Where is the triangle?]