Math 224 **Summer 2019**
Quiz 4 **Drew Armstrong**

Problem 1. Let X be the continuous random variable defined by the following pdf:

$$f(x) := \begin{cases} 2x & 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute the mean $\mu = E[X]$.

$$E[X] = \int_0^1 x \cdot f(x)\,dx = \int_0^1 2x^2\,dx = 2 \cdot \frac{x^3}{3}\bigg|_0^1 = \frac{2}{3}$$

(b) Compute the variance $\sigma^2 = \mathrm{Var}(X) = E[X^2] - E[X]^2$.

First we compute the second moment:

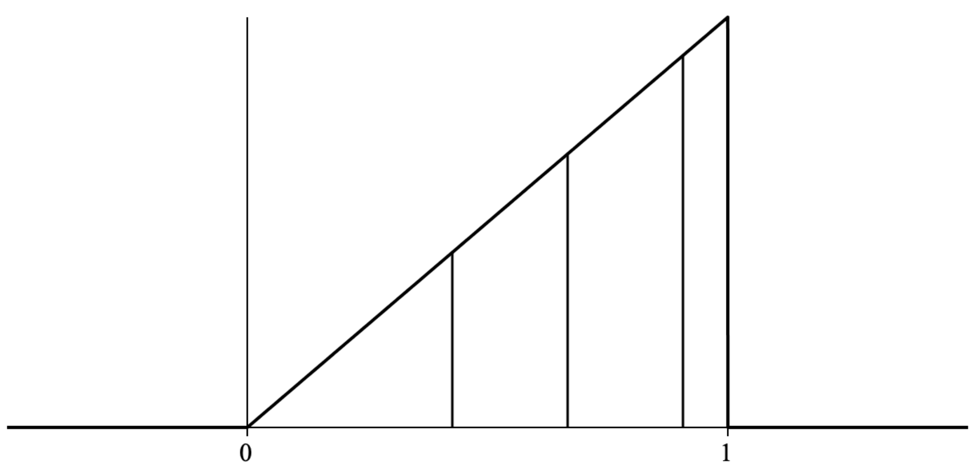
$$E[X^2] = \int_0^1 x^2 \cdot f(x)\,dx = \int_0^1 2x^3\,dx = 2 \cdot \frac{x^4}{4}\bigg|_0^1 = \frac{1}{2}$$

Then we compute the variance:

$$\sigma^2 = \mathrm{Var}(X) = E[X^2] - E[X]^2 = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{18}.$$

(c) Draw the graph of $f(x)$, labeled with μ and σ. [Estimate the value of σ.]

Note that the standard deviation is $\sigma = \sqrt{1/18} \approx 0.24$. Here is the picture:

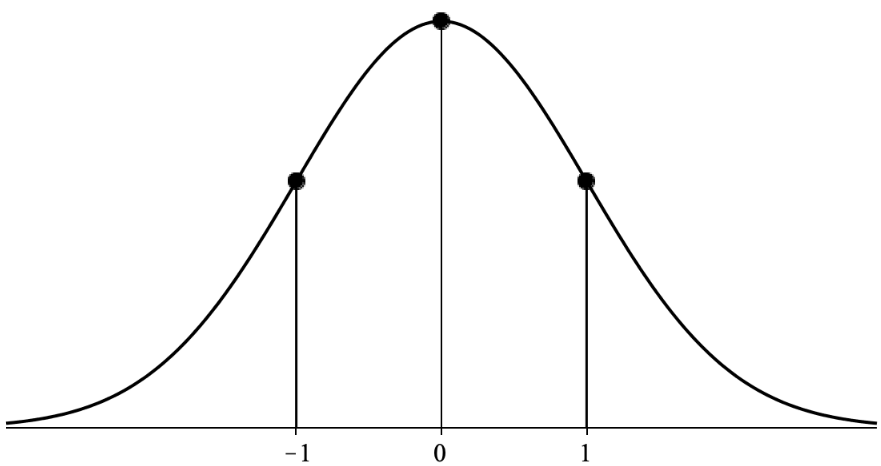


Problem 2. Let Z be a standard normal random variable (i.e., with $\mu = 0$ and $\sigma = 1$).

(a) Tell me the probability density function:

$$f_Z(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$$

(b) Sketch the graph of $f_Z(x)$, showing the maximum and the points of inflection.



(c) Use the attached table to compute the probability $P(-0.25 < Z < 0.5)$.

$$\begin{aligned} P(-0.25 < Z < 0.5) &= \Phi(0.5) - \Phi(-0.25) \\ &= \Phi(0.5) - (1 - \Phi(0.25)) \\ &= \Phi(0.5) + \Phi(0.25) - 1 \\ &= (0.6915) + (0.5987) - 1 \\ &= 29.02\%. \end{aligned}$$