Math 224 **Summer 2019**
Quiz 2 **Drew Armstrong**

Problem 1. A fair 6-sided die has 3 sides painted red, 2 sides painted green and 1 side painted blue. Suppose you roll the die 4 times and let R, G, B denote the number of times you get red, green, blue, respectively.

(a) Compute $P(R = 3, G = 1, B = 0)$.

$$P(R = 3, G = 1, B = 0) = \frac{4!}{3!1!0!}\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^1\left(\frac{1}{6}\right)^0 = \frac{1}{6}.$$

(b) Compute $P(R = 4, G = 0, B = 0)$.

$$P(R = 4, G = 0, B = 0) = \frac{4!}{4!0!0!}\left(\frac{3}{6}\right)^4\left(\frac{2}{6}\right)^0\left(\frac{1}{6}\right)^0 = \frac{1}{16}.$$

(c) Compute $P(B \geq 1)$.

We think of the die as a coin with $P(\text{blue}) = 1/6$ and $P(\text{not blue}) = 5/6$. Then

$$P(B \geq 1) = 1 - P(B = 0) = 1 - P(\text{not blue})^4 = 1 - \left(\frac{5}{6}\right)^4 = 51.8\%.$$

Problem 2. An urn contains 3 red balls, 2 green balls and 1 blue ball. You reach in and grab 4 balls (without replacement). Let R, G, B denote the number of red, green, blue balls that you get, respectively.

(a) Compute $P(R = 3, G = 1, B = 0)$.

$$P(R = 2, G = 1, B = 1) = \frac{\binom{3}{3}\binom{2}{1}\binom{1}{0}}{\binom{6}{4}} = \frac{2}{15}.$$

(b) Compute $P(R = 4, G = 0, B = 0)$.

Since there are only 3 red balls it is **impossible** to get 4. If we write $\binom{3}{4} = 0$ then the general formula gives the correct answer:

$$P(R = 4, G = 0, B = 0) = \frac{\binom{3}{4}\binom{2}{0}\binom{1}{0}}{\binom{6}{4}} = 0.$$

(c) Compute $P(B \geq 1)$. [Hint: There is only 1 blue ball in the urn.]

We will treat the balls as "blue" and "not blue." Since there is 1 blue ball and 5 non-blue balls we have the following possibilities:

$$P(B = 0) = \frac{\binom{1}{0}\binom{5}{4}}{\binom{6}{4}} = \frac{1}{3},$$
$$P(B = 1) = \frac{\binom{1}{1}\binom{5}{3}}{\binom{6}{4}} = \frac{2}{3}.$$

We conclude that

$$P(B \geq 1) = P(B = 1) = \frac{2}{3}.$$