Math 224 **Summer 2019**
Quiz 1 **Drew Armstrong**

Problem 1. Let S be a sample space of **twelve equally likely outcomes**: $\#S = 12$. Now consider two events $A, B \subseteq S$ such that

$$\#A = 7, \qquad \#B = 5 \qquad \text{and} \qquad \#(A \cup B) = 9.$$

(a) Find the number of outcomes in the intersection: $\#(A \cap B)$.

$$\begin{aligned} \#(A \cup B) &= \#A + \#B - \#(A \cap B) \\ \#(A \cap B) &= \#A + \#B - \#(A \cup B) \\ \#(A \cup B) &= 7 + 5 - 9 = \boxed{3} \end{aligned}$$

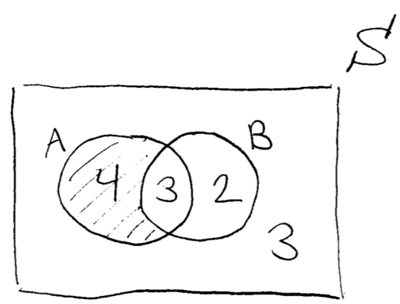
(b) Compute the probability that "A and B both happen."

Since the outcomes are equally likely we have

$$P(A \cap B) = \frac{\#(A \cap B)}{\#S} = \boxed{\frac{3}{12}} = 25\%.$$

(c) Compute the probability that "A happens and B does **not** happen."

A Venn diagram can help with this:



Or we we can use the formula $\#A = \#(A \cap B) + \#(A \cap B')$. Either way we find that $\#(A \cap B') = 4$ and hence

$$P(A \cap B') = \frac{\#(A \cap B')}{\#S} = \boxed{\frac{4}{12}} = 33.3\%.$$

Problem 2. A coin is flipped 4 times in sequence. Compute the following probabilities, assuming that $P(H) = 1/3$ and $P(T) = 2/3$.

(a) The probability of getting the sequence $HTHT$.

Since the coin flips are independent we have

$$P(HTHT) = P(H)P(T)P(H)P(T) = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \boxed{\frac{4}{81}} = 4.9\%.$$

(b) The probability of getting "exactly 2 heads."

The number of ways to get 2 heads is $\binom{4}{2} = 6$. If X is the number of heads then

$$P(X = 2) = \binom{4}{2} P(H)^2 P(T)^2 = 6 \cdot \frac{4}{81} = \boxed{\frac{24}{81}} = 29.6\%.$$

(c) The probability of getting "at least 1 head."

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \left(\frac{2}{3}\right)^4 = \frac{81}{81} - \frac{16}{81} = \boxed{\frac{65}{81}} = 80.2\%.$$