Math 224 **Fall 2021**
Homework 1 **Drew Armstrong**

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1. Suppose that a fair coin is flipped 5 times in sequence and let X be the number of "heads" that show up. Draw Pascal's triangle down to the sixth row (recall that the zeroth row consists of a single 1) and use your table to compute the probabilities $P(X = k)$ for $k = 0, 1, 2, 3, 4, 5$.

2. Suppose that a fair coin is flipped 4 times in sequence.
(a) List all 16 outcomes in the sample space S.
(b) List the outcomes in each of the following events:
$A = \{\text{at least 1 head}\}$,
$B = \{\text{more than 2 heads}\}$,
$C = \{\text{heads on the 1st flip, anything on the other flips}\}$,
$D = \{\text{heads on the 1st and 2nd flips, anything on the other flips}\}$.
(c) Assuming that all outcomes are **equally likely**, use the formula $P(E) = \#E/\#S$ to compute the following probabilities:

$$P(A \cup B), \quad P(A \cap B), \quad P(C), \quad P(D), \quad P(C \cap D).$$

3. Draw Venn diagrams to verify *de Morgan's laws*: For all events $A, B \subseteq S$ we have
(a) $(A \cup B)' = A' \cap B'$,
(b) $(A \cap B)' = A' \cup B'$.

4. Let $A, B \subseteq S$ be two events satisfying $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$. Use this information to compute the following probabilities. A Venn diagram may be helpful.
(a) $P(A \cup B)$,
(b) $P(A \cap B')$,
(c) $P(A \cup B')$.

5. Suppose that you roll a pair of **fair** six-sided dice. For the sake of argument, let's suppose that one die is blue and the other is red, so we can tell the dice apart.
(a) Write down all elements of the sample space S. What is $\#S$?
(b) Compute the probability of getting a "double six", i.e., a six on each die. [Hint: Let $E \subseteq S$ be the set of outcomes that correspond to "double six". What is $\#E$? Assuming that all outcomes are equally likely, you can use the formula $P(E) = \#E/\#S$.]

6. Consider a strange coin with $P(H) = 1/3$ and $P(T) = 2/3$. Suppose that you flip the coin 5 times and let X be the number of heads that you get. Find the probability $P(X \leq 4)$. [Hint: Observe that $P(X \leq 4) + P(X = 5) = 1$. Maybe it's easier to compute $P(X = 5)$.]

7. Analyze the Chevalier de Méré's two experiments:
(a) Roll a **fair** six-sided die 4 times and let X be the number of "sixes" that you get. Compute $P(X \geq 1)$. [Hint: You can think of a die roll as a "strange coin flip", where $H =$"six" and $T =$"not six".]
(b) Roll a pair of **fair** six-sided dice 24 times and let Y be the number of "double sixes" that you get. Compute $P(Y \geq 1)$. [Hint: You can think of one roll of the dice as a "very strange coin flip", where $H =$"double six" and $T =$"not double six".]

[Hint: Problems 5 and 6 are relevant.]