Math 224 | **Fall 2017**
Homework 3 | **Drew Armstrong**

Problems from 9th edition of *Probability and Statistical Inference* by Hogg, Tanis and Zimmerman:

- Section 2.1, Exercises 6, 7, 8, 12.
- Section 2.3, Exercises 1, 3, 4, 12, 13, 14.
- Section 2.4, Exercises 12.

Additional Problems.

1. Two Formulas for Expectation. Let S be the sample space of an experiment (assume that S is finite) and let $X : S \to \mathbf{R}$ be any random variable. Let $S_X \subseteq \mathbb{R}$ be the "support" of X, i.e., the set of possible values that X can take. Explain why the following formula is true:

$$\sum_{s\in S} X(s)\cdot P(s) = \sum_{k\in S_X} k\cdot P(X=k).$$

2. Expected Value of a Binomial Random Variable.

(a) Use the explicit formula for binomial coefficients to prove that

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

(b) Use part (a) to compute the expected value of a binomial random variable:

$$\begin{aligned} E[X] &= \sum_{k=0}^{n} k\binom{n}{k}p^k(1-p)^{n-k} \\ &= \sum_{k=1}^{n} k\binom{n}{k}p^k(1-p)^{n-k} \\ &= \sum_{k=1}^{n} n\binom{n-1}{k-1}p^k(1-p)^{n-k} \\ &= \cdots \end{aligned}$$