**Math 211** | **Summer 2022**
**Algebraic Properties** | **Drew Armstrong**

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**Vectors.** Let $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{0} \in \mathbb{R}^n$ and $r, s \in \mathbb{R}$.

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

$\mathbf{u} + \mathbf{0} = \mathbf{u}$

$r(s\mathbf{u}) = (rs)\mathbf{u}$

$(r + s)\mathbf{u} = r\mathbf{u} + s\mathbf{u}$

$r(\mathbf{u} + \mathbf{v}) = r\mathbf{u} + r\mathbf{v}$

$1\mathbf{u} = \mathbf{u}$

$0\mathbf{u} = \mathbf{0}$

**Dot Product.** Let $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and $c \in \mathbb{R}$.

$\mathbf{u} \bullet \mathbf{v} = \mathbf{v} \bullet \mathbf{u}$

$\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$

$c(\mathbf{u} \bullet \mathbf{v}) = (c\mathbf{u}) \bullet \mathbf{v} = \mathbf{u} \bullet (c\mathbf{v})$

$\mathbf{u} \bullet \mathbf{u} = \|\mathbf{u}\|^2$

$\|\mathbf{u}\| = 0$ if and only if $\mathbf{u} = \mathbf{0}$

$\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos\theta$ (angle measured tail to tail)

$\mathbf{u} \bullet \mathbf{v} = 0$ if and only if $\mathbf{u}$ and $\mathbf{v}$ are perpendicular

**Cross Product.** Let $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$ and $c \in \mathbb{R}$.

$\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$

$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$

$c(\mathbf{u} \times \mathbf{v}) = (c\mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (c\mathbf{v})$

$\mathbf{u} \times \mathbf{0} = \mathbf{0}$

$\mathbf{u} \times \mathbf{u} = \mathbf{0}$

$\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\| \sin\theta$ (angle measured tail to tail, $0 \leq \theta < \pi$)

**Derivatives.** Let $\mathbf{u}, \mathbf{v} : \mathbb{R} \to \mathbb{R}^n$, $f : \mathbb{R} \to \mathbb{R}$, and $c \in \mathbb{R}$.

$[c\mathbf{u}(t)]' = c\mathbf{u}'(t)$

$[\mathbf{u}(t) \pm \mathbf{v}(t)]' = \mathbf{u}'(t) \pm \mathbf{v}'(t)$

$[f(t)\mathbf{u}(t)]' = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$

$[\mathbf{u}(t) \bullet \mathbf{v}(t)]' = \mathbf{u}'(t) \bullet \mathbf{v}(t) + \mathbf{u}(t) \bullet \mathbf{v}'(t)$

$[\mathbf{u}(t) \times \mathbf{v}(t)]' = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}'(t) \times \mathbf{v}(t)$ (here we need $n = 3$)

$[\mathbf{u}(f(t))]' = \mathbf{u}'(f(t))f(t)$ (this is a vector $\mathbf{u}'(f(t))$ times a scalar $f(t)$)