Math 211 **Summer 2022**
Homework 1 **Drew Armstrong**

Problem 1. Lines and Circles. For each curve compute the velocity vector and the speed at time t. Also eliminate t to find an equation relating x and y.

(a) $(x, y) = (a + ut, b + vt)$ where a, b, u, v are constants.
(b) $(x, y) = (a + r\cos t, b + r\sin t)$ where a, b, r are constants.

Problem 2. An Interesting Curve. Consider the parametrized curve

$$(x, y) = (t^2 - 1, t^3 - t).$$

(a) Eliminate t to find an equation relating x and y. [Hint: Note that $y/x = t$.]
(b) Find the points on the curve where the tangent line is vertical, horizontal, or has slope $+1$ or -1. [Hint: The slope of the tangent at time t is $dy/dx = (dy/dt)/(dx/dt)$.]
(c) Use the information in part (b) to sketch the curve.

Problem 3. The Cycloid. The cycloid is an interesting curve whose arc length can be computed by hand. It is parametrized by

$$(x, y) = (t - \sin t, 1 - \cos t).$$

(a) Sketch the curve between times $t = 0$ and $t = 2\pi$. [Hint: The slope of the tangent line at time t is $(dy/dt)/(dx/dt) = \sin t/(1 - \cos t)$, which goes to $+\infty$ as $t \to 0$ from the right and goes to $-\infty$ as $t \to 2\pi$ from the left. You do not need to prove this.]
(b) Compute the arc length between $t = 0$ and $t = 2\pi$. [Hint: You will need the trig identities $\sin^2 t + \cos^2 t = 1$ and $1 - \cos t = 2\sin^2(t/2)$.]

Problem 4. A Triangle in Space. Consider the following points in $\mathbb{R}^3$:

$$P = (1, 1, -1), \quad Q = (1, -1, 1), \quad R = (-1, 1, 1).$$

(a) Find the coordinates of the three side vectors $\mathbf{u} = \vec{PQ}, \mathbf{v} = \vec{QR}, \mathbf{w} = \vec{PR}$.
(b) Use the length formula to compute the three side lengths $\|\mathbf{u}\|, \|\mathbf{v}\|, \|\mathbf{w}\|$.
(c) Use the dot product to compute the three angles of the triangle.

Problem 5. Some Vector Arithmetic. Let $\mathbf{u}$ and $\mathbf{v}$ be any two vectors, living in 527-dimensional space. Use the rules of vector arithmetic (pages 112 and 147) to show that

$$\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(\mathbf{u} \bullet \mathbf{v}).$$

[Hint: Start with $\|\mathbf{u} - \mathbf{v}\|^2 = (\mathbf{u} - \mathbf{v}) \bullet (\mathbf{u} - \mathbf{v})$. Now use FOIL and simplify the result.]

Problem 6. Equations of Lines and Planes. The equation of the line in $\mathbb{R}^2$ that contains the point (x_0, y_0) and is perpendicular to the vector $\mathbf{n} = \langle a, b\rangle$ is

$$a(x - x_0) + b(y - y_0) = 0.$$

The equation of the plane in $\mathbb{R}^3$ that contains the point (x_0, y_0, z_0) and is perpendicular to the vector $\mathbf{n} = \langle a, b, c\rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

(a) Find the equation of the line containing $(2, 0)$ and perpendicular to $\langle 4, 3\rangle$.
(b) Find the equation of the plane containing $(1, 0, 0)$ and perpendicular to $\langle 1, 1, 1\rangle$.